EXPERIMENTAL ANALYSIS OF DOPPLER EFFECT USING THE ROTATING SOUND EMITTING SOURCE

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Compiled August 27, 2019

While the principle to observe the Doppler Effect with a rotating source is known and very simple, there are no existing commercial apparatus for laboratory exercise. The full realization of the apparatus and an easy procedure to study Doppler Effect in the laboratory practice is described through a paper.

Key words: Doppler effect, rotation.

PACS: 01.30.-y, 01.55.+b, 01.40.-d

1. INTRODUCTION

Doppler Effect was discovered by Christian Johann Doppler in 1842. A short biographical survey can be found in reference [1]. Doppler Effect is manifested as a frequency shift of the wave when source and/or receiver are moving. Doppler shift has significant importance for astrophysical measurements [2]. It provides observations of the source velocities regardless of their distances. Doppler shift measurements also enable to deduce velocities in the fluids what is very useful in a medicine and engineering [3]. In meteorology, the vertical velocities of clouds can be determined using Doppler effect [4]. Doppler broadening of spectral lines are useful tool for various measurements in experimental physics: temperature of hot plasma [5], electronic and defect structures in solids (positron annihilation spectroscopy) [6], etc. Numerous examples indicate needs to familiarize students with Doppler Effect experimentally in general physics courses.

Even nowadays, after more than hundred years since one of the first proposed classroom experiment [7], Doppler Effect experiments are still lack in practice. Modern improvisations for Doppler Effect demonstrations holds on the same idea [8]. To study Doppler Effect through quantitative measurements, the wave signals of translatory moving, vibrating or rotating sources are analyzed using PC sound card and appropriate software [9–11]. However, described apparatus are temporary realized and non-practical in every day lab practice. Devices intended to study acoustic Doppler
Effect also can be irritated because of the high volume of the sound source. Observations of Doppler Effect on the terrain are also suggested [12].

One more promotion of an easy to use, non-temporary apparatus for study acoustic Doppler Effect could contribute to better quality of laboratory practice. While the data collection implies using the commercial software, such as Sound Forge, the procedure remained very simple because data acquisition is carried out online. Detailed theoretical and experimental analysis is demonstrated and suggested as a standard laboratory practice for study Doppler Effect at undergraduate level.

2. DESCRIPTION OF THE APPARATUS

![Fig. 1 – Apparatus for demonstration of Doppler effect.](image)

Apparatus realized for demonstration of Doppler Effect is presented in Fig. 1. The apparatus is consisted of rotating disk made of Plexiglas mounted on the steel axis with a ball bearings hitched to a transparent housing made of 5mm Plexiglas and plywood. To minimize standing waves and echoes, inner part of the housing was coated with a sound dampening material. The sound source with adjustable frequency and two serial connected Li-ion rechargeable batteries are installed on the rotating disk. The source we made from IC NE555 [13] and a high quality headphone. Battery charge connector, trimmer potentiometer for frequency adjustment and a fuse are positioned on the rotating disk and are available through the little hole made on the front side of the housing. Switch of the sound source placed on the rotating disk can be turned on/off (when rotation is stopped) via a rubber button situated on the front side of the housing.

Condenser microphone, assembled on the outer part of the housing and con-
nected to the PC, is fixed along a horizontal line with the sound source in the highest position. Sound waves reaching the microphone through a 2cm diameter whole made on the wooden part of housing. Disk rotation is belt driven by a DC motor connected to adjustable voltage supply. The switch for turning on rotation clockwise is positioned at the front side of housing. Safety fuse for DC motor drive and the voltage supply connectors are positioned at the rear side of the housing.

3. THEORETICAL ANALYSIS

If the sound source at the moment $t = 0$ is in the highest point (Fig. 2.a), than the current distance of the sound source from the receiver (microphone) is given by

$$s(t) = \sqrt{(d - r \sin \omega t)^2 + r^2(1 - \cos \omega t)^2},$$

where $d$ denotes horizontal distance of the receiver from axis of rotational disk, $r$ is radial distance of the source from rotating axis, $\omega$ is angular velocity and $t$ is a moment of time. Current speed of the sound emitter relative to the sound receiver is obtained as a derivative of the eq. (1) with respect to time

$$v(t) = \frac{ds}{dt} = \omega r \frac{r \sin \omega t - d \cos \omega t}{\sqrt{d^2 - 2dr \sin \omega t + 2r^2(1 - \cos \omega t)}}.$$

Keeping in mind that the sound source is moving around a circle, during the one revolution sound source achieve maximal approaching and departing speed

$$v_{max} = \omega r = \frac{2\pi r}{T},$$

where $T$ is a period of rotation. Maximum approaching and departing speed of the sound source fits to the positions when a direction connecting the source and receiver is a tangent line on the circle determined by the rotating source (Fig. 2.b).

To determine maximum approaching and departing speed of the sound source, according to eq. (3) one has to measure a period of rotation and the radial distance of the sound source. Period of rotation can be determined in the sound analysis software by determination of the time interval between the two most intensive parts of the wave signal.

According to Doppler equation, minimum frequency received by microphone is

$$\nu_{min} = \nu_0 \frac{\nu_0}{\nu_0 + v_{max}},$$

where $\nu_0$ is the frequency of sound emitter, and $\nu_0$ is the sound speed. Maximum frequency registered by microphone during one revolution is

$$\nu_{max} = \nu_0 \frac{\nu_0}{\nu_0 - v_{max}}.$$
Dividing eq. (5) by eq. (4), and after some simple transformations follows the equation for experimental determination of the sound speed.

\[ v_0 = \frac{v_{\text{max}} + v_{\text{min}}}{v_{\text{max}} - v_{\text{min}}}. \]  

(6)

The minimum and maximum received frequency can be also determined in the sound analysis software using a spectrum analysis view.

Reciprocal values of the minimum and maximum frequencies are

\[ \frac{1}{\nu_{\text{min}}} = \frac{1}{\nu_0} \frac{\nu_0 + \nu_{\text{max}}}{\nu_{\text{max}}}, \]  

(7)

\[ \frac{1}{\nu_{\text{max}}} = \frac{1}{\nu_0} \frac{\nu_0 - \nu_{\text{max}}}{\nu_{\text{max}}}. \]  

(8)

Summation of eqs. (7) and (8), and simple transformations leads to the equation for calculation the sound emitter frequency

\[ \nu_0 = 2 \frac{\nu_{\text{max}} \cdot \nu_{\text{min}}}{\nu_{\text{max}} + \nu_{\text{min}}}. \]  

(9)

From eq. (9) follows that emitted frequency is given by the harmonic mean of the maximum and minimum received frequency.

Advanced analysis of the received sound can be conducted by measuring the frequency as a function of time. If the maximum velocity of the sound emitter in eq. (4) is replaced by relation for current speed of the emitter, eq. (2), follows

\[ \nu(t) = \nu_0 \frac{v_0}{v_0 + v(t)}. \]  

(10)

When the frequency as a function of time is analysed, the time interval \( \Delta t \) between the minimum and maximum received frequency is an important quantity. It
can be obtained graphically considering Fig. 2.b. From simple trigonometry follows:
\[
\sqrt{d^2 + r^2 \cos \left( \frac{\omega t}{2} \right)} = r.
\] (11)

Here the time moment \( t \) corresponds to the time interval \( \Delta t \) which follows from eq. (11):
\[
\Delta t = \frac{2}{\omega} \arccos \left( \frac{r}{\sqrt{d^2 + r^2}} \right).
\] (12)

4. EXPERIMENTAL RESULTS

Sound waves received by the attached microphone are analysed using the commercial software Sound Forge 7. The microphone was placed at the horizontal distance of \( d = 20\,\text{cm} \) from the highest point of the sound source. Doppler effect was studied using the sound emitter at frequency of \( \nu_0 = 4809\,\text{Hz} \). Volume of the source is set to optimal value-silent outside apparatus and minimal battery consumption (approximately 36 hours of continuous work without recharging the batteries). The source was turned on, and for each voltage presented in Table 1, the sound waves were recorded into 5 seconds lasting frames in a single file. Analyzing the intensities of the sound in each frame, the periods of rotations are obtained. Velocities of the sound emitter corresponding to radial distance of \( r = 0, 145\,\text{m} \) are calculated using eq. (3). However, when a function of velocity versus voltage is established, this part of the exercise in repeated experiments can be omitted.

The frequencies of received sound were observed in the online spectrum analysis window as presented on the Fig. 3. Before starting rotation of the source, sound emitter frequency is observed directly from the spectrum view. Minimum and maximum frequencies are determined in real time as the lowest and highest stable peaks in the spectrum (See Fig. 3) for each radial velocity.

Using the experimental results of the registered frequencies and the eqs (6) and (9), the velocity of sound and the frequency of source are calculated. The results are summarized in Table 1.

Advanced analysis of the registered frequency as a function of time was conducted for a voltage of 7\text{V}. The ”sonogram” view of the Sound Forge software allows obtaining the experimental data of the received frequency at the moment of time. The noise from environment, DC motor, belt drive and ball bearings is minimized using the digital Equalizer incorporated in the software.

Experimental data fitted with the function given by eq. (10), where \( v(t) \) is replaced by eq. (2), are presented in the Fig. 4. Time delay was not taken into account because the peripheral speed was much lower than the speed of a sound. The time interval between the minimum and the maximum frequency observed from Fig.
Fig. 3 – Spectrum of the sound source: in a stable position; rotating with a peripheral speed of $v_{\text{max}} = 7.43 \text{ m/s}$.

Fig. 4 – Received frequency as a function of time at voltage $U = 7V$. 
Table 1.
Experimental results.

<table>
<thead>
<tr>
<th>U [V]</th>
<th>T [s]</th>
<th>n [rpm]</th>
<th>$v_{max}[\frac{m}{s}]$</th>
<th>$v_{min}[Hz]$</th>
<th>$v_{max}[Hz]$</th>
<th>$v_0[\frac{m}{s}]$</th>
<th>$v_0[Hz]$</th>
</tr>
</thead>
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<tr>
<td>6</td>
<td>0.152333</td>
<td>394</td>
<td>5.98</td>
<td>4732</td>
<td>4898</td>
<td>347</td>
<td>4814</td>
</tr>
<tr>
<td>7</td>
<td>0.122667</td>
<td>489</td>
<td>7.43</td>
<td>4709</td>
<td>4922</td>
<td>336</td>
<td>4813</td>
</tr>
<tr>
<td>8</td>
<td>0.104333</td>
<td>575</td>
<td>8.73</td>
<td>4693</td>
<td>4940</td>
<td>341</td>
<td>4813</td>
</tr>
<tr>
<td>9</td>
<td>0.092667</td>
<td>647</td>
<td>9.83</td>
<td>4679</td>
<td>4955</td>
<td>343</td>
<td>4813</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>6.84</td>
<td>7.28</td>
<td>4798</td>
<td>4925</td>
<td>344</td>
<td>4813</td>
</tr>
</tbody>
</table>

4 is: $\Delta t = 35.4 ms$. The same quantity was calculated using eq. (11), which yields: $\Delta t = 36.8 ms$.

5. CONCLUSION

Practical realization of the non-temporary apparatus with rotating source and using the Sound Forge software enable us to study Doppler Effect simply. Sound wave analyses provide the results necessary to measure the sound speed and to verify Doppler equation calculating and directly observing the frequency of sound emitter. Described apparatus is successfully tested during one semester with the group of about 400 students.

Based on our lab practice, it seems that rotating source is preferable over translational. There are at least two reasons: Rotating source in safety housing is an easy to handle and the effect can be observed on-line quantitatively during a longer period of time with a sound analysis software. The apparatus we developed is not sensitive to environmental noise, what is very important where relatively large number of students do exercises simultaneously.

Acknowledgements. We gratefully acknowledge lab technician Pavle Petković for very useful suggestions.

REFERENCES
13. 555 timerIC (Wikibook).