DETERMINATION HALF-LIVE OF HEAVY NUCLEI USING FERMI GAS MODEL

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Abstract. Calculations on the α-decay half-lives of heavy nuclei are performed through Fermi gas model. The Wood Saxon potential is employed to calculate half-lives through a Coulomb barrier. The present study is initially restricted to even-even nuclei in the heavy mass region with $N > 126$. Then the study is extended to the heaviest nuclei including super heavy elements. The main consideration in this model is $V_0b^2 = 100$ MeV, where $b$ is the force range between two nucleons which represents the attractive well, and $-V_0$ is depth. The calculated α-decay half lives are found to be in good agreement with the experimental data for heavy nuclei with approximately spherical shape. The results are independent from deformation parameters for super heavy nuclei.

Key words: Fermi gas model, heavy and super-heavy nuclei, half lives.

1. INTRODUCTION

The study of alpha decay dates back to the early days of nuclear physics, even to the first observation of unknown radiation by Becquerel in 1896. With the foundation and development of quantum mechanics, Gamow [1], Condon and Gurney [2] independently described α-decay as a quantum tunneling problem for the first time in 1928. These pioneering works were the first successful applications of quantum mechanics to nuclear physics. Different from the cluster model based on the Gamow picture, some other theoretical models, such as the shell model and fission like model, have also been proposed in the pursuit of a microscopic description of α-decay. Consequently, the absolute α-decay width has been estimated by many theoretical calculations [3–17], which employ various approaches such as WKB method [4–7], coupled channel approach [14, 15] and phenomenological methods [16, 17]. On the experimental side, as an engaging topic in contemporary nuclear physics, the observation of α-decay chains from
unknown parent nuclei to known nuclei has been a reliable method used to identify different super heavy elements (SHEs) and isometric states [18–21]. Moreover, α-decay, as one of the most important decay modes for unstable nuclei, has been a useful and precise tool in the investigation of nuclear structure for a long time. In the present research, the Fermi gas model is employed to calculate α-decay half-lives for heavy and super heavy nuclei. The features of this nuclear model has been used to solve the Schrödinger equation numerically to evaluate wave functions of alpha particles at attractive nuclear potential and the beyond region of the Coulomb barrier between alpha particle and the rest of nuclei as daughter nuclei.

2. THEORETICAL FRAMEWORK

The Fermi Gas model has two approaches about the parts of potential, including nuclear attraction at $r < R$ and Coulomb potential at $r \geq R$, for which the exclusion Pauli principle is considered [23, 27]. Following sections reveal the details of calculations in this article.

2.1. DEPTH AND WIDTH FOR NUCLEAR POTENTIAL WELL

In our attempt to account for some nuclear properties with Fermi gas model as simple and crude model, we should concentrate initially on the parameters empirically observed in the Weizsacker formula. Consider a system of a number of neutrons and protons put in a cubic box with linear dimension. The Schrödinger equation for a single particle in this box reduces to

$$\frac{\hbar^2}{2M} \nabla^2 \psi = E \psi. \quad (1)$$

The solution of equation (1) with boundary conditions is given by

$$\psi(x, y, z) = A \sin k_x \cdot \sin k_y \cdot \sin k_z \cdot z, \quad (2)$$

where $A$ is a normalization factor and

$$k_x a = n_x \pi, \quad k_y a = n_y \pi \quad \text{and} \quad k_z a = n_z \pi, \quad (3)$$

where $n_x$, $n_y$, and $n_z$ are all positive integers [23]. Each set of positive integers $(n_x, n_y, n_z)$ defines a different solution corresponding to an energy

$$E(n_x, n_y, n_z) = \frac{\hbar^2}{2M} (k_x^2 + k_y^2 + k_z^2) = \frac{\hbar^2}{2M} k^2. \quad (4)$$

It is obvious from relation (4) as well as equation (2) that $k = (k_x, k_y, k_z)$ is the momentum of the particle in the box. Because of the Pauli principle, a given
momentum state can be occupied mostly by four nucleons. The number of permissible solutions $n(k)$ with the magnitude of $k$ between $k$ and $k + dk$ is given by

$$dn(k) = \frac{1}{8} 4\pi k^2 dk \frac{1}{(2\pi)^3}$$

(5)

afterwards, the highest occupied momentum state, $k_f$, will be given according to relation (6)

$$\frac{A}{4} = \frac{4\pi}{3} \frac{k_f^3}{8(\pi/a)^3} \quad \text{or} \quad A = \frac{2\Omega}{3\pi^2} k_f^3$$

(6)

where $\Omega = a^3$ is the volume of the box. Under these conditions, the momentum of the highest occupied state depends only on the density $\rho = A/\Omega$ of nucleons in the box, and is given by

$$\rho = \frac{2}{3\pi^2} k_f^3.$$  

(7)

Thus far the interaction among the nucleons is disregarded. Within this limit the momentum distribution per unit volume in momentum space is a step function with a constant value for $k < k_f$ and zero for $k > k_f$. This momentum distribution is referred to as the Fermi distribution. From the observed density of nuclei, $\rho = 1.72 \times 10^{38} \text{ particle/cm}^3$, which, as known, is practically the same for all nuclei with $A \geq 12$, we obtain [23],

$$k_f = 1.36 \text{ fm}^{-1} \text{ with corresponding energy of } \varepsilon_f = 38 \text{ MeV}.$$  

(8)

$k_f$ and $\varepsilon_f$ are called the Fermi momentum and Fermi energy of the degenerate gas model respectively [23]. To calculate the depth of nuclear well potential according to Fermi gas model and Bethe Weizsacker semi empirical mass formula, we can consider the binding energy per nucleon which is added to Fermi energy, as

$$V_0 = \varepsilon_f + B/A.$$  

(9)

Since heavy and super-heavy nuclei have surplus of neutrons, the Fermi level of the protons and neutrons in a stable nucleus has to be equal. The increase in kinetic energy with density is easy to understand if we recall the Pauli repulsion among nucleons. Because of this repulsion, an increase in density is accompanied by an increase in the density of nodes in the wave function and therefore an increase in its curvature. Therefore the average curvature is of the order of magnitude of distance between the nucleons, that is $\rho^{-1/3}$. The kinetic energy is quadratic in the inverse curvature of the wave function and, hence, the factor $\rho^{2/3}$.

To obtain an estimation of the equilibrium density of nuclei, we have to know also the density dependence of the potential energy. We can estimate this in
the following way. Take the interaction between two nucleons to be represented by an attractive square well of range \(b\) and depth \(-V_0\) (\(V_0 > 0\)). At a given density, there is a probability, \(p\), for a nucleon to be the range of forces of other prescribed nucleons. The total contribution to the potential energy will, therefore, be

\[
<V> = \frac{-A(A-1)}{2} pV_0
\] (10)

The probability, \(p\), can be estimated for large nuclei, to the extent that we can neglect surface effects. It is just the ratio of the interaction volume to the total volume, that is

\[
p = \frac{\frac{4}{3} \pi b^3}{\Omega} = \frac{4}{3} \pi \frac{1}{A} b^3 \rho \quad \text{for} \quad \Omega \gg b^3.
\] (11)

It seems that this conclusion is in disagreement with the experimental data. Nuclei are experimentally known to have more or less a constant density, but certainly not a constant radius. Additionally, their binding energy is roughly proportional to \(A\), but certainly not proportional to \(A^2\). We must take into account the repulsive parts of the nuclear interaction rather than the attractive square well adopted for the nucleon–nucleon interaction. The equilibrium density observed in nuclei must be the combined effects of the Pauli repulsion, the dominant nucleon-nucleon attraction, and the particular features of the nucleon-nucleon repulsion. As the kinetic energy is proportional to \(\rho^{2/3}\) or inversely proportional to \(r_0^2\), \(2r_0\) is the average spacing between nucleons. If \(r_c\) is the radius of the repulsive core and the space between the nucleons equals \(r_c\), then the kinetic energy should go to infinity. For the average potential energy, we take the relation (11) with \(p\) again for finite nuclei. Noting that \(R = r_0A^{1/3}\), we obtain the expression for the average potential energy

\[
<V> = -V_0 \frac{A}{2} \left(\frac{b}{r_0}\right)^2 \left[1 - \frac{9}{16} \frac{b}{R} + \frac{1}{32} \left(\frac{b}{R}\right)^3\right].
\] (12)

To obtain the equilibrium density, the \(<T> + <V>\) has to minimize with respect to \(r_0\)

\[
V_0 b^2 = \frac{4\alpha}{2 \left(\frac{b}{r_0}\right)^3 \left[1 - \frac{3}{4} \frac{b}{r_0\alpha^2} + \frac{3}{16} \left(\frac{b}{r_0\alpha^2}\right)\right] \left[1 - \left(\frac{b}{r_0}\right)^3\right].
\] (13)

The \(V_0 b^2\) is known from the low energy scattering data and a good approximation is \(V_0 b^2 = 100\) MeV (fm)\(^2\). On the basis of Fermi gas model, this relation is the main condition to determine the width of nuclear potential well [23].
2.2. COULOMB ENERGY (C.E) in FERMI GAS MODEL

At the liquid drop model, nucleus to be considered as a uniformly charged sphere of charge $Ze$ and charge density $\rho = \frac{Ze}{4\pi R^3}$. The electrical energy of the nucleus is therefore [28]

$$V_{\text{coulomb}} = \int_0^R \frac{4}{3} \pi R^3 \rho 4\pi r^2 \, dr \, \rho \frac{1}{r} = \frac{3}{5} \frac{Z^2 e^2}{R}.$$  \hspace{1cm} (14)

In Fermi gas model approach the charge density is the function of charge radius and thickness of nucleus. Initially this function has approximately a trapezoidal shape from the function is drawn. In this approach the amount of thickness approximately equal to 3.5 fm. As the quoted empirical value of the surface thickness, the distance between the radii at which $\rho = 0.9$ and 0.1 of the central value, the thickness is predicted, according to trapezoidal distribution, 2.8 fm (Fig. 2).

![Trapezoidal charge distribution](image)

Fig. 2 – The trapezoidal charge distribution assumed in deriving surface thickness.

The effect of the Pauli repulsion and Pauli attraction is illustrated by the simple but important example of the Coulomb energy for nuclear ground state in reference [23]. According to

$$C.E = \frac{A(A-1)}{2} \int \rho_{00}(r_1, r_2) \, \psi_{12}(r_1, r_2) \, dr_1 \, dr_2,$$  \hspace{1cm} (15)

where $\psi_{12}$ is given by

$$\psi_{12} = \frac{e^2}{r_{12}} \left[ 1 + \frac{r_1(1)}{2} \right] \left[ 1 + \frac{r_2(2)}{2} \right] \rho_{12} \equiv |r_1 - r_2|.$$
the isospin factors guarantee that the interaction takes place only between protons. By changing variables from \( r_1 \) and \( r_2 \) to \( r \) (\( \equiv r_1 - r_2 \)) and evaluating the integral, it becomes

\[
(C. E)_{ex} = \frac{9\pi Z^2(2-1)e^2}{4\Omega k_f^2} \left[ 1 - j_0^2(k_f R) - j_1^2(k_f R) \right].
\] (16)

where \( j_0 \) and \( j_1 \) are the spherical Bessel function. These spherical Bessel function terms can be dropped as \( k_f R \) becomes large. Hence a very good approximation is

\[
(C. E)_{ex} = \frac{9\pi Z^2e^2}{4\Omega k_f^2}.
\] (17)

The total Coulomb energy is then

\[
C. E = \frac{3Z^2e^2}{5} - (C. E)_{ex} \rightarrow C. E = \frac{0.7772}{A^2} \left( 1 - \frac{1.09}{A^2} \right) \text{ [MeV]}. \] (18)

For achievement to realistic charge distribution issue the Wood-Saxon function of charge distribution is utilized in calculations, substituted by what is considered in

\[
\rho(r) = \frac{\rho_0}{1 + \exp \left( \frac{r - \bar{R}}{a} \right)}.
\] (19)

where \( \bar{R} \) is the average of radius and \( a \) is the diffuseness coefficient with constant amount equal to 0.54 fm [24]. Thus, the total radius of nucleus, \( R_{total} \), can be calculated by accumulation of average radius, \( \bar{R} \) and the thickness of nucleus with

\[
R_{total} = \bar{R} + 2a \ln 3.
\] (20)

The thickness is considered as the change of nuclear charge distribution from 90% initial value to 10% of it [30]. The rms of radius is then conveniently written as

\[
\bar{R} = < r^2 >^{1/2} = \left[ \int \rho(r)^2 r^2 dr \right]^{1/2}.
\] (21)

For large \( A \), relation (18) should be compared with the semi empirical result of Myers and Swiatecki [24], whom considered Fermi (Wood-Saxon), distribution as charge distribution in nucleus (Fig. 3)

\[
C. E = \frac{0.7122}{A^{1/3}} \left( 1 - \frac{1.69}{A^{2/3}} \right) \text{ MeV}.
\] (22)
In this article all Coulomb energies of alpha-decay nuclei are calculated by relation (22) which is modified by adding the effect of thickness and changing the charge distribution function from trapezoidal shape in relation (18) to Wood-Saxon.

There is no remarkable difference between charge distribution functions for calculation alpha decay half lives for heavy nuclei while the Wood-Saxon function could emerge results which are significantly close to experimental data. Since all even–even nuclei and their isotopes are in $0^+$ situations, the centrifugal term is eliminated from calculations.

2.3. NUMERICAL SOLUTION OF SCHRODINGER EQUATION

The half life for radioactive decay of the heavy and super-heavy nuclei can be computed by numerical solution of the Schrödinger equation, by “finite difference method” in MATLAB software [29]. The concept of this method is the approximation of the second derivative by the difference formula

$$\frac{d^2\psi(x)}{dx^2} = \frac{\psi(x+\Delta x)-2\psi(x)+\psi(x-\Delta x)}{\Delta x^2},$$  (23)

The Schrödinger equation is

$$\frac{d^2\psi(x)}{dx^2} = -k(x)^2\psi(x),$$

where

$$k(x) = \sqrt{\frac{2m}{\hbar^2}(E - U(x))}.$$  (24)

In the finite difference method, it could be started with
\[ x = [x(1), x(2), ..., x(N)] \quad k = [k(1), k(2), ..., k(N)], \]  
\hfill (25.a)

where \( N \) is the maximum number of \( x \) coordinates, \( x(1) = x_{\text{min}} \) and \( x(N) = x_{\text{max}} \), therefore

\[ \psi(x_{\text{min}}) = \psi(x(1)) = 0 \text{ and } \psi(x(2)) = \psi(x(1) + \Delta x) = 1. \]  
\hfill (25.b)

Then as \( x \) is incremented, the other values of \( \psi(x) \) are calculated from the equation

\[ \psi(x_{c+1}) = ((2 - (k_c \Delta x)^2)\psi(x_c) - \psi(x_{c-1}) \text{ for } c = 2 \text{ to } N - 1. \]  
\hfill (25.c)

When a physically accepted solution is found for \( n^{th} \) state, the wave function is normalized by numerically integrating the wave function using Simpson rule, so

\[ \int_{x(1)}^{x(N)} |\psi(x)|^2 \, dx = A_n \rightarrow \psi_n(x) = \frac{\psi(x)}{\sqrt{A_n}}. \]  
\hfill (25.d)

In Schrödinger equation, energy \( E \) is given by \( Q_\alpha \), the energy of alpha particle, and potential energy \( U(x) \) is given by

\[ U(x) = \begin{cases} \frac{-V_0}{r} & r < b \\ \frac{ZZ' e^2}{r} & r > b \end{cases}. \]  
\hfill (26)

Moreover, for numerical solution of relation (24), it is necessary to determine dimensions of nuclear well potential and the Coulomb energy \( C.E \) of potential barrier which could be found in section 2.1 and 2.2. To determine the \( Q_\alpha \), it is necessary to notice tunneling process of alpha particle through the Coulomb barrier. In this case, two turning points are shown in Fig. 4. The first one, \( b \), is on the edge of nuclear well potential where the particle proceeds to tunneling and the second turning point, \( b' \), is in the point of escaping particle from the Coulomb barrier. So the relation (27) determines the value of \( Q_\alpha \) as;

\[ C.E = \frac{ZZ e^2}{b} \rightarrow ZZ' e^2 = b \times C.E \]  
\hfill (27.a)

\[ V(b') = Q_\alpha = \frac{ZZ' e^2}{b'} \]  
\hfill (27.b)

\[ Q_\alpha = \frac{b \times (C.E)}{b'}. \]  
\hfill (27.c)
By using relations (27), (22) and (20) we can determine the first input parameter to solve the Schrödinger equation numerically. To calculate the half life for each nucleus, it is necessary to determine mass number, $A$, and atomic number, $Z$, for the parent nucleus in addition to the $Q_n$.

![Fig. 4 – Alpha particle tunneling through the Coulomb barrier.](image)

The relation to calculate alpha decay half life of nuclei is

$$t_{1/2} = \frac{\ln 2}{fP}. \quad (28)$$

In relation (28), $f$ is frequency and is given by

$$f = \frac{V_\alpha}{2b} \quad (29)$$

$V_\alpha$ is velocity of alpha particle between turning points and given by

$$V_\alpha = \sqrt{\frac{2Q_\alpha}{m_\alpha}}. \quad (30)$$

and the probability, $P$, is given by,

$$P = \left(\frac{A_{out}}{A_{in}}\right)^2. \quad (31)$$
where, $A_{in}$ and $A_{out}$ are amplitudes of wave functions inside the potential well and outside the well respectively [31]. The Schrödinger equation must be solved for a potential energy function which represents the system of the parent nucleus and the alpha particle to find amplitudes.

3. RESULT AND DISCUSSION

After calculating alpha decay half life in even–even states for heavy nuclei, it is realized that there is a good agreement between theoretical assumptions on the basis of Fermi gas model and experimental data [34, 35], just for heavy nuclei, having an approximately spherical shapes and a small amount of deformation $\beta$ parameter, including Quadrupole deformation, with respect to heavier isotopes in each group of heavy nuclei. The higher deviation between theoretical and experimental data, the higher $\beta$ deformation parameter in isotopes. For example one can see good agreement between theoretical and experimental data for alpha decay half life for $^{232}_{92}$U with $\beta_2 = 0.008$, whereas by turning to heavier isotopes, the deformation parameter gets rise to $\beta_2 = 0.241$ for $^{238}_{92}$U, and the discrepancy between Fermi gas model calculation results and empirical data is obvious. In Po isotopes, the nuclear deformation is changing from oblate shape for the lightest isotopes into a rather spherical shape for the heaviest isotopes, so the discrepancy between theoretical calculations and experimental data is extremely likely due to the effect of shape coexistence [33]. Furthermore, due to similarity treatment between the isotopes at the end of Table 1, for example No and Rf isotopes, and the super heavy nuclei, one can see the gradual independency of alpha decay half life to the deformation parameters.

As the extension of calculations for super-heavy nuclei, the results show that there is a good agreement between the data concerning to theoretical calculation on the basis of Fermi gas model and experimental data [36–39]. The analysis of super-heavy nuclei data correlation, shown in Table 2, illustrates the independency of theoretical calculations from deformation $\beta$ parameters in various SHNs. Therefore, it can produce the reliability of Fermi gas model application for super heavy nuclei structure to study as a crude and simple nuclear model in comparison with the others like Shell model and Nilsson model.
Table 1
Experimental and calculated α-decay half lives for even-even nuclei
with neutron number \( N > 126 \)

<table>
<thead>
<tr>
<th>Element</th>
<th>4</th>
<th>Z</th>
<th>( B.E ) (MeV)</th>
<th>( U_d ) (MeV)</th>
<th>( t_{(fm)} )</th>
<th>( E_{c} ) (MeV)</th>
<th>( R_{\alpha \text{eff}} ) (μm)</th>
<th>( E_{d} ) (MeV)</th>
<th>( \beta_{2} )</th>
<th>( (T_{1/2})_{\text{cal}} ) (S)</th>
<th>( (T_{1/2})_{\text{exp}} ) (S)</th>
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<tbody>
<tr>
<td>Po</td>
<td>212</td>
<td>84</td>
<td>1655.70</td>
<td>45.80</td>
<td>1.477</td>
<td>790.98</td>
<td>7.08</td>
<td>164.98</td>
<td>0.055</td>
<td>2.99 \times 10^{-7}</td>
<td>3.08 \times 10^{-7}</td>
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<tr>
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<td>84</td>
<td>1666.02</td>
<td>45.78</td>
<td>1.477</td>
<td>786.61</td>
<td>7.09</td>
<td>164.28</td>
<td>0.022</td>
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<td>1.80 \times 10^{-4}</td>
</tr>
<tr>
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<td>216</td>
<td>84</td>
<td>1675.91</td>
<td>45.75</td>
<td>1.478</td>
<td>786.41</td>
<td>7.11</td>
<td>163.13</td>
<td>0.000</td>
<td>1.45 \times 10^{-1}</td>
<td>3.88 \times 10^{-7}</td>
</tr>
<tr>
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<td>218</td>
<td>84</td>
<td>1685.40</td>
<td>45.73</td>
<td>1.478</td>
<td>784.24</td>
<td>7.13</td>
<td>162.24</td>
<td>0.009</td>
<td>1.86 \times 10^{-2}</td>
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<td>826.84</td>
<td>7.09</td>
<td>171.92</td>
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<td>813.66</td>
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<td>1.480</td>
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<td>1735.10</td>
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<td>809.91</td>
<td>7.20</td>
<td>164.06</td>
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<td>92</td>
<td>1695.50</td>
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<td>7.16</td>
<td>192.92</td>
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Table 1

(continued)

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α-decay half lives for even-even heavy nuclei are calculated on the basis of Fermi gas model approaches, and the results are in a good agreement with experimental data for spherical shape of heavy nuclei. Extending the calculation for SHNs, the results are in accordance with empirical data without any considerations for deformation parameters. This result shows that the Fermi gas model could be applied to study the structure of nuclei in SHNs and as a simple nuclear model among other nuclear structure models, it has the potential to be extended to further investigation of α-decay properties.

**Acknowledgments.** The author would like to thank dr. Ion Cooper and prof. Dorin Poenaru for numerous fruitful discussions on this problem.

---

1 The same calculation is done for SHE with $l = (5/2)^+$, though centrifugal potential is considered in calculation and results are in good agreement with half life for these nuclei.
REFERENCES