INTERROGATION METHODS FOR BRAGG FIBER-BASED PLASMONIC SENSORS

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Abstract. The angular and spectral interrogation methods are applied to calculation of the reflectivity, power loss, and spectral and amplitude sensitivities for the TE, TM, and hybrid modes in a Bragg fiber-based plasmonic sensor with four layers. The results are in agreement with the recent studies of the same structure by using another analytical method, where the electromagnetic field is represented by a Bessel function of the first kind in the core region (SiO$_2$), a linear combination of Bessel functions of the first and second kinds in the dielectric interior layer (GaP), a linear combination of the Hankel functions in the gold region, and a modified Bessel function of the second kind in the outermost region (H$_2$O).

Key words: sensors, Bragg fiber, surface plasmon resonance, photonic band gap

1. INTRODUCTION

During the past few years there was a large interest in studies of plasmonic sensors [1-26] with applications in chemical, biological, and medical sciences. Thus, some plasmonic biosensors [13, 19-20] are applied for detection of human blood groups, and of hemoglobin concentration in human blood and human liver tissues.

The interrogation (angular or spectral) method has been used for the analysis of fiber based plasmonic sensors [22-24] and hollow core Bragg optical fiber [25]. In a recent paper [10] one uses an analytical method, where a linear combination of the Hankel functions $H_1$ and $H_2$ represent the field in the gold region of a fiber-based plasmonic sensor. This method was applied for a structure with three, four and five layers [7-9]. When the analyte is the distilled water, the difference between the resonant wavelengths calculated with the finite element method and with the analytical method is very small (0.19 nm for four layers).

In this paper the angular and spectral interrogation methods are applied to a Bragg optical fiber-based plasmonic sensor when the dispersions of SiO$_2$, GaP, gold and distilled water (H$_2$O) are considered. The power loss and amplitude sensitivity are increased at optimized thicknesses of the GaP and gold layers for the TE, TM, and hybrid mode. The results are in agreement with the recent studies of the same structure by using another analytical method [9] where the field is represented by a Bessel function of the first kind in the core region (SiO$_2$), a linear
combination of Bessel functions of the first and second kinds in the dielectric interior layers (GaP), a linear combination of the Hankel functions in the gold region, and a modified Bessel function of the second kind in the external sensing medium (H₂O).

2. BRAGG FIBER WITH FOUR LAYERS

Figure 1 shows a Bragg fiber with four layers where \( n_1, n_2, n_3, \) and \( n_4 \) are the refractive indices of the core (SiO₂), GaP, gold, and H₂O, respectively. The thicknesses of the GaP and gold layers are \( d_2 \) and \( d_3 \), respectively. Figure 2 shows a contour plot of the z-component \( S_z(x, y) \) of the Poynting vector for the \( TE \) and \( TM \) modes at a nonresonant wavelength. The light is incident under the angle \( \alpha \) from an air medium in a SiO₂ core of the fiber and the angle inside the fiber is \( \theta \). The relation between these angles (Fig. 1) is given by the Snell’s law:

\[
\frac{\sin \alpha}{\sin(90 - \theta)} = \frac{n_1}{n_{air}},
\]

where \( n_{air} = 1 \) is the refractive index of the air.

The refractive index of the SiO₂ [22], GaP [27], and distilled water [28] materials are calculated through a Sellmeier-type relation. The refractive index of the gold layer is calculated by the Drude model [29] as in references [7-10].

3. ADAPTED INTERROGATION METHODS

In the angular interrogation method [24-25], the wavelength is kept constant and the angle of incidence is varied and a sharp dip (maximum of the power loss)
appears at a resonance incidence angle $\theta$ in the reflectivity. In the spectral interrogation method, the angle of incidence is kept constant and the wavelength is varied. A sharp dip (maximum of the power loss) appears at a wavelength $\lambda$ in the reflectivity. The resonance incidence angle and the resonance wavelength are dependent on the refractive index of the sensing medium.

![Contour plots](image)

**Fig. 2** - Contour plots of the z-component $S_z(x, y)$ of the Poynting vector for the $TE$ and $TM$ modes at a nonresonant wavelength. The arrows indicate the orientation of the electric field for these modes.

![Contour plots](image)

**Fig. 3** - Contour plot of the z-component $S_z(x, y)$ of the Poynting vector at the resonance ($\lambda = 0.6805 \, \mu m$) between the core-guided $TM$ (a) and plasmon $TM$ (b) modes for a fiber with four layers made by SiO$_2$ core (radius $r_1 = 1.527 \, \mu m$) surrounded by a GaP layer (thickness $d_2 = 40 \, nm$), a gold layer (thickness $d_3 = 40 \, nm$), and by a water layer.
The power loss (in dB) for a \( TM \) mode for a Bragg fiber with \( N = 4 \) layers is \([25-26]\):

\[
PL(TM) = 10 \log \left( \frac{1}{P(TM)} \right),
\]

and the output power \( P(TM) \) is:

\[
P(TM) = R(TM) \frac{L}{D \tan(\theta)},
\]

where \( R(TM) \) is the corresponding intensity reflection coefficient, \( L \) is the sensing length, \( D \) is the fiber core diameter, and \( \theta \) is the angle inside the fiber \((L/D = 25)\). Here

\[
R(TM) = |r(TM)|^2,
\]

where the amplitude reflection coefficient is:

\[
r(TM) = \frac{(M_{11m} + M_{12m}q_{11m})q_{1m} - (M_{21m} + M_{22m}q_{4m})}{(M_{11m} + M_{12m}q_{11m})q_{1m} + (M_{21m} + M_{22m}q_{4m})}.
\]

In the above relation we have

\[
M_m = M_{2m}M_{3m} = \begin{pmatrix} M_{11m} & M_{12m} \\ M_{21m} & M_{22m} \end{pmatrix},
\]

where

\[
M_{2m} = \begin{pmatrix} \cos(\beta_2) & -i \sin(\beta_2) \\ -iq_{2m} \sin(\beta_2) & \cos(\beta_2) \end{pmatrix},
\]

\[
M_{3m} = \begin{pmatrix} \cos(\beta_3) & -i \sin(\beta_3) \\ -iq_{3m} \sin(\beta_3) & \cos(\beta_3) \end{pmatrix},
\]

\[
q_{1m} = \frac{\sqrt{n_1^2 - n_m^2 \sin^2(\theta)}}{n_1}, \quad q_{2m} = \frac{\sqrt{n_2^2 - n_m^2 \sin^2(\theta)}}{n_2},
\]

\[
q_{3m} = \frac{\sqrt{n_3^2 - n_m^2 \sin^2(\theta)}}{n_3}, \quad q_{4m} = \frac{\sqrt{n_4^2 - n_m^2 \sin^2(\theta)}}{n_4},
\]
\[ \beta_2 = \frac{2\pi d_2}{\lambda} \sqrt{n_2^2 - n_1^2 \sin^2(\theta)}, \beta_3 = \frac{2\pi d_3}{\lambda} \sqrt{n_3^2 - n_1^2 \sin^2(\theta)}. \] (10)

Fig. 4 - Reflectivity (a) and power loss (b) versus the angle \( \alpha \) for the core modes \( TM \) of a Bragg fiber \( (N = 4, d_1 = 40 \text{ nm}) \) at the wavelength \( \lambda = 0.62909 \mu\text{m} \).

Fig. 5 - Reflectivity (a), (b), (c) and corresponding power loss (d), (e), (f) versus wavelength for the core modes \( TE, TM, \text{ and } TE + TM \) for \( \alpha = 12.8028^\circ \) \( (\Theta = 81.2526^\circ) \), near the wavelength \( \lambda = 0.62909 \mu\text{m} \).

4. NUMERICAL RESULTS AND DISCUSSION

Figure 3 shows the contour plot of the z-component \( S_z(x, y) \) of the Poynting vector at the resonance \( (\lambda = 0.6805 \mu\text{m}) \) between the core-guided \( TM \) and plasmon \( TM \) modes for a fiber with four layers made by SiO\(_2\) core \( (r_1=1.527 \mu\text{m}) \) surrounded by a GaP layer \( (d_2 = 40 \text{ nm}) \), a gold layer \( (d_3 = 40 \text{ nm}) \), and by a water layer. Figure 4 shows the reflectivity and power loss versus the incident angle \( \alpha \) on the core layer \( (\text{SiO}_2) \) for \( TM \) mode at
\[ \lambda = 0.62909 \ \mu m \] where \( PL = 127.0848 \text{dB}, \ SNR = 0.21, \) and \( \text{FOM} = 209.5 \text{RIU}^{-1}. \) The power loss is increased when the reflectivity is decreased.

Figure 5 shows the reflectivity and the corresponding power loss versus wavelength for the core modes \( TE, TM \) and \( TE + TM \) for \( \alpha = 12.8028^o \) (\( \theta = 81.2526^o \)), near the wavelength \( \lambda = 0.62909 \ \mu m. \) The minimum reflectivity is at the same wavelength as the corresponding maximum power loss for a given mode. For a \( TM \) mode, the shift towards longer wavelengths of the loss matching point for an increase \( \Delta n_a \) of the analyte refractive index by 0.001 RIU is \( \delta \lambda_{\text{res}} = 2.89 \ \text{nm}, \) the full width at half maximum (FWHM) of the loss spectra is \( \delta \lambda_{0.5} = 12.5 \ \text{nm}, \) the signal-to-noise ratio is \( \text{SNR} = 0.23, \) the figure of merit is \( \text{FOM} = 230.7 \text{RIU}^{-1}, \) the maximum value of the power loss is \( \text{PL} = 128.2 \ \text{dB}, \) the angle inside the fiber is \( \theta = 81.2526^o, \) the wavelength is \( \lambda = 0.629328 \ \mu m \) and the difference between maximal amplitude sensitivity and resonant wavelengths is \( \Delta \lambda_\lambda = 3.5 \ \text{nm}. \)

Figure 6 shows the amplitude sensitivity versus wavelength for the core modes \( TE, TM \) and \( TE + TM \) for \( \alpha = 12.8028^o \) (\( \theta = 81.2526^o \)) near the wavelength \( \lambda = 0.62909 \ \mu m. \)

Figure 7 shows the reflectivity and power loss versus the angle \( \alpha \) for the core modes \( TM \) of a Bragg fiber \( (N = 4, d_2 = 40 \ \text{nm}) \) at the wavelength \( \lambda = 0.6805 \ \mu m. \)

Figure 7 shows the reflectivity and power loss versus the angle \( \alpha \) for the core modes \( TM \) of a Bragg fiber \( (N = 4, d_2 = 40 \ \text{nm}) \) at the wavelength \( \lambda = 0.6805 \ \mu m. \)
It is interesting to note that for the incident angle $\alpha = 18.8565^{\circ}$ on the core layer ($\text{SiO}_2$) and for the corresponding angle $\theta = 77.1725^{\circ}$ inside the fiber at the same wavelength ($\lambda = 0.6805 \mu m$) one obtains the same values for the power loss ($PL = 116.9938 \text{ dB}$), signal-to-noise ratio $SNR = 0.11$, and figure of merit $FOM = 112.4 \text{ RIU}^{-1}$, but different values for the shifts $\Delta \alpha_{res}$ and $\Delta \theta_{res}$ for an increase $\Delta n_a$ of the analyte refractive index by 0.001 RIU and the full width at half maximum ($FWHM$) of the angular values ($\Delta \alpha_{res} = 0.3311^{\circ}$ and $FWHM = 2.9447^{\circ}$ for $\alpha$; $\Delta \theta_{res} = 0.2209^{\circ}$ and $FWHM = 1.9630^{\circ}$ for $\theta$).

Figure 8 shows the reflectivity, power loss, and amplitude sensitivity versus wavelength for the core modes $TE$, $TM$, and $TE + TM$ for $\theta = 77.1725^{\circ}$, near the wavelength $\lambda = 0.6805 \mu m$. For a $TM$ mode, the shift towards longer wavelengths of the loss matching point for an increase $\Delta n_a$ of the analyte refractive index by 0.001 RIU is $\delta \lambda_{res} = 3.94 \text{ nm}$, the full width at half maximum ($FWHM$) of the loss spectra is $\delta \lambda_{0.5} = 35.8 \text{ nm}$, the signal-to-noise ratio is $SNR = 0.11$, the figure of merit is $FOM = 110.1 \text{ RIU}^{-1}$, the maximum of the amplitude sensitivity is $S_A = 203.1 \text{ RIU}^{-1}$, the maximum value of the power loss is $PL = 116.99 \text{ dB}$, the angle inside the fiber is $\theta = 77.1725^{\circ}$, the wavelength is $\lambda = 0.680493 \mu m$ and the difference between maximal amplitude sensitivity and resonant wavelengths is $\Delta \lambda_A = 8.95 \text{ nm}$. It is important that real part of the effective index $Re (\beta / k) = n_1 \sin \theta = 1.4557514 \sin(77.1725^{\circ}) = 1.419420$ is close to $Re (\beta / k) = 1.421346$ from the analytical method.

For $\lambda = 0.62909 \mu m$ and $d_2 = d_3 = 40 \text{ nm}$, $r_1 = 1.530 \mu m$, the effective index $\beta / k$ of the core and plasmonic mode $HE_{11}$ are $1.446532 + 0.00352789i$ and $1.4496993 + 0.00356700$, respectively. For the same values of $\lambda$, $d_2$, and $d_3$, but $r_1 = 1.531 \mu m$, the effective index $\beta / k$ of the core and plasmonic mode $HE_{11}$ are $1.446545 + 0.00354053i$ and $1.4496985 + 0.00355251$, respectively. For the same values of $\lambda$, $d_2$, and $d_3$, but $r_1 = 1.532 \mu m$, the effective index $\beta / k$ of the core and plasmonic mode $HE_{11}$ are $1.4496979 + 0.00353795i$ and $1.4465572 + 0.00355323i$, respectively. One observe that for $r_1 = 1.531 \mu m$, there is a loss matching point at
\( \lambda = 0.62909 \, \mu m \), where the difference between the imaginary parts of the effective indices of the core and plasmon modes is minimum (0.0000119861).

Figure 9 shows that the real and imaginary parts of the effective index for the core \( TM_{01} \) mode are increasing with the radius \( r_1 \) of the core layer. Thus for \( r_1 = 1.527 \, \mu m \), \( \beta/k = 1.432556 + 0.001523i \), and for \( r_1 = 1.800 \, \mu m \), \( \beta/k = 1.438243 + 0.002178i \) when \( \lambda = 0.62909 \, \mu m \) and \( d_2 = d_3 = 40 \, nm \). In the angular interrogation method, the radius of the core layer is absent (but is assumed large) in the calculation of the reflectivity. Thus for \( \lambda = 0.62909 \, \mu m \), \( r_1 = 1.527 \, \mu m \), \( r_2 = 1.567 \, \mu m \) and \( r_3 = 1.607 \, \mu m \), the real part of the effective index is \( \text{Re} \left( \frac{\beta}{k} \right) = n_1 \sin \Theta = 1.440198 \), where \( n_1 = 1.457126 \) and \( \Theta = 81.2264^\circ \).

Figure 10 shows the reflectivity, power loss and amplitude sensitivity versus the thickness of the gold layer \( (d = d_3) \) and of GaP layer \( (d = d_2) \) for the core \( TE \), \( TM \), and hybrid \( TE + TM \) modes of a Bragg fiber \( (N = 4) \) for the angle \( \Theta = 77.1725^\circ \) at the wavelength \( \lambda = 0.6805 \, \mu m \). The optimum thicknesses to obtain the minimum reflectivity (the maximum power loss) are: \( d_2 = 40 \, nm \), \( d_3 = 42 \, nm \) when \( \text{PL} = 149.4 \, dB \) or \( d_2 = 39 \, nm \), \( d_3 = 40 \, nm \) when \( \text{PL} = 117.0 \, dB \) for the \( TM \) mode, \( d_2 = 40 \, nm \), \( d_3 = 68 \, nm \) when \( \text{PL} = 6.0 \, dB \) or \( d_2 = 34 \, nm \), \( d_3 = 40 \, nm \) when \( \text{PL} = 18.9 \, dB \) for the \( TE \) mode and \( d_2 = 40 \, nm \), \( d_3 = 42 \, nm \) when \( \text{PL} = 22.6 \, dB \) or \( d_2 = 34 \, nm \), \( d_3 = 40 \, nm \) when \( \text{PL} = 35.0 \, dB \) for the hybrid \( TE + TM \) mode. The better values for \( \text{PL} \) are obtained if for fixed values of the thickness \( d_2 \), \( d_3 \), and \( \Theta \) one evaluates the \( \text{PL} \) versus the wavelength \( \lambda \). Thus, for a \( TM \) mode with \( d_2 = 40 \, nm \), \( d_3 = 42 \, nm \) and \( \Theta = 77.1725^\circ \), the maximum value of \( \text{PL} = 192.6 \, dB \) is obtained for \( \lambda = 0.678113 \, \mu m \), i.e. at a shorter wavelength in agreement with the behaviour of
$HE_{11}$ mode in the analytical method [9] for an increase of the thickness $d_3$ of the gold layer.

In this case the full width at half maximum (FWHM) of the loss spectra is $\delta \lambda_{0.5} = 15.2$ nm, the shift $\delta \lambda_{res} = 3.9$ nm towards longer wavelengths of the loss matching point for an increase $\Delta \eta_a$ of the analyte refractive index by 0.001 RIU, the signal-to-noise ratio $SNR = 0.26$, the figure of merit $FOM = 258.3$ RIU$^{-1}$, the maximum of the amplitude sensitivity $S_A = 617.9$ RIU$^{-1}$ and the difference $\Delta \lambda_A = 4.2$ nm between maximal amplitude sensitivity and resonant wavelengths. For the finite element method [9] we get $\delta \lambda_{res} = 3.2$ nm, $\delta \lambda_{0.5} = 19.4$ nm, $SNR = 0.17$, $FOM = 164.9$ RIU$^{-1}$, and $\Delta \lambda_A = 2.1$ nm for a $HE_{11}$ mode when $r_1 = 1.527$ µm, $r_2 = 1.567$ µm, $r_3 = 1.607$ µm ($d_2 = d_3 = 40$ nm) and $\lambda = 0.62909$ µm. The loss matching point ($\lambda = 0.6293$ µm) and the effective index for the core ($\beta/k = 1.449669 + 0.00354217i$) and plasmon ($\beta/k = 1.446438 + 0.00355322i$) modes for the mode $HE_{11}$ are calculated by using an analytical method with a step of 0.0001 in the wavelength range. For the same structure [9], the analytical method gives a loss matching point for the core $TM_{01}$ mode at a higher wavelength ($\lambda = 0.6805$ µm) and the effective indices for the core and plasmon modes are $\beta/k = 1.421346 + 0.00341024i$ and $\beta/k = 1.438907 + 0.00341046i$, respectively. In this case, $\delta \lambda_{res} = 6.0$ nm (686.5 nm - 680.5 nm), $\delta \lambda_{0.5} = 99.5$ nm, $SNR = 0.06$, $FOM = 60.3$ RIU$^{-1}$ and the power loss (2735.0 dB/cm) for the core $TM_{01}$ mode is very large. It is interesting to note that if in the angular method one replace $L/D = 25$ with $L/D = 355$ where $L$ is the sensing length and $D$ is the fiber core diameter one obtains a power loss $PL = 2735.2$ dB at the same wavelength ($\lambda = 0.678113$ µm) as for $L/D = 25$, but the maximum of the amplitude sensitivity is unchanged ($S_A = 617.9$ RIU$^{-1}$ at $\lambda = 0.68234$ µm).
Table 1 shows the shift $\delta \lambda_{\text{res}}$ towards longer wavelengths of the loss matching point for an increase $\Delta n_a$ of the analyte refractive index by 0.001 RIU, the full width at half maximum (FWHM) of the loss spectra $\delta \lambda_{0.5}$, the signal-to-noise ratio $\text{SNR}$, the figure of merit $\text{FOM}$, the maximum of the amplitude sensitivity $S_A$, the maximum value of the power loss $PL$, the angle inside the fiber $\Theta$, the wavelength $\lambda$, and the difference $\Delta \lambda_A$ between maximal amplitude sensitivity and resonant wavelengths.

Table 1
Values of $\delta \lambda_{\text{res}}$ [nm], $\delta \lambda_{0.5}$ [nm], $\text{SNR}$, $\text{FOM}$ [RIU$^{-1}$], $S_A$ [RIU$^{-1}$], $PL$ [dB], $\Theta$ [deg], $\lambda$ [\text{m}], and $\Delta \lambda_A$ [nm] for a $TM$ mode of an optical fiber with four layers with the thicknesses $d_2$ [nm] and $d_3$ [nm] of the GaP and gold layers, respectively.

<table>
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<th>$d_2$; $d_3$</th>
<th>$\delta \lambda_{\text{res}}$</th>
<th>$\delta \lambda_{0.5}$</th>
<th>$\text{SNR}$</th>
<th>$\text{FOM}$</th>
<th>$S_A$</th>
<th>$\lambda$</th>
<th>$\Delta \lambda_A$</th>
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<td>0.23</td>
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<td>413.1</td>
<td>128.2</td>
<td>81.2526</td>
</tr>
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<td>35.8</td>
<td>0.11</td>
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<td>203.1</td>
<td>117.0</td>
<td>77.1725</td>
</tr>
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<td>15.2</td>
<td>0.26</td>
<td>258.3</td>
<td>617.9</td>
<td>192.6</td>
<td>77.1725</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

The results obtained by using the interrogation method are in agreement with the recent studies of the same structure by using another analytical method [7], where the electromagnetic field is based on the Bessel functions. Thus, for a $TM$ mode with $d_2 = 40\text{nm}$, $d_3 = 42 \text{ nm}$ and $\Theta = 77.1725^\circ$, the maximum value of $PL = 192.6 \text{ dB}$ is obtained for $\lambda = 0.678113 \text{ \mu m}$, i.e. at a shorter wavelength in agreement with the behavior of the $HE_{11}$ mode in the analytical method [9] for an increase of the thickness $d_3$ of the gold layer. Also, the amplitude sensitivity for the core modes near the maximum power loss point for the Bragg fiber with a gold layer is increased for optimized thicknesses of the GaP and gold layers.

REFERENCES