EFFECTS OF PERIODICALLY-MODULATED THIRD-ORDER DISPERSION ON PERIODIC SOLUTIONS OF NONLINEAR SCHRÖDINGER EQUATION WITH COMPLEX POTENTIAL

BIN LIU¹, LU LI¹,*, DUMITRU MIHALACHE²

¹Institute of Theoretical Physics, State Key Laboratory of Quantum Optics and Quantum Optics Devices, Shanxi University, Taiyuan 030006, China
*Corresponding author, Email: llz@sxu.edu.cn
²Horia Hulubei National Institute of Physics and Nuclear Engineering, Reactorului 30, Magurele, Bucharest, Romania

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Abstract. We study, both analytically and numerically, families of periodic solutions of the nonlinear Schrödinger equation with periodically-modulated third-order dispersion (TOD) and complex-valued potential. The TOD and the complex potential are built as solutions of an inverse problem, which predicts the explicit expressions of the complex-valued potential and the TOD supporting a required phase-gradient structure of the periodic solutions. We investigate in detail the band structure of the stability problem of the periodic solutions in the corresponding periodic complex potential by means of plane-wave-expansion method and direct numerical simulations of the evolution of the perturbed inputs. The results show that the band stability domains of the periodic solutions may be narrowed when increasing the nonlinear effects and the amplitudes of periodic solutions.

Key words: Stability problem band structure; Periodic solution; Plane-wave-expansion method; Third-order dispersion; Complex potential.

1. INTRODUCTION

Theoretical and experimental studies in the area of \( \mathcal{PT} \)-symmetric, non-Hermitian physical systems have received considerable attention over the past two decades [1–4]. \( \mathcal{PT} \)-symmetric Hamiltonians form a noteworthy class of a non-Hermitian system that has a balanced energy exchange with its surroundings; see the earlier review paper [4] and recent overviews [5–8]. It was shown in the pioneering paper by Bender and Boettcher [1] that a non-Hermitian physical system with unbroken \( \mathcal{PT} \)-symmetry has an entirely real spectrum.

Due to the equivalence between quantum mechanical Schrödinger equation and optical wave equation, the concept of \( \mathcal{PT} \)-symmetry was extended to classical optical systems with complex-valued external potentials by use of optical amplification
(gain) and absorption (loss) [9]. Especially, realizations of $PT$-symmetric potentials in optics attracted a great interest of the researchers. They exhibit remarkable properties and potential applications [10–16], such as power oscillation [10], non-reciprocal light propagation [17], optical transparency [18], negative refraction [19], pseudo-Hermitian Bloch oscillation [20, 21], unidirectional invisibility [22–24], coherent perfect laser absorption [25], and possibilities to design various $PT$-symmetric devices [26–28]. Thus, optics provides an especially successful platform to investigate the unique dynamics in $PT$-symmetric systems, including the formation of bright and dark solitons, gap solitons, defect solitons, lattice solitons, vortices, all-optical signal control etc. [5, 6, 29–63].

Besides, the concept of $PT$-symmetry has been also applied to Bose-Einstein condensates [64–67] (for a recent overview on Bose-Einstein condensation and related phenomena, see Ref. [68]), atomic cells [69–71], and nonlinearity-induced $PT$-symmetry without material gain [72].

Among those investigations, stability is a key issue and has attracted great attention from the researchers. In fact, the stability of optical solitons and nonlinear beam dynamics in non-Hermitian potentials was addressed, showing that the solitons may be stable in a wide range of parameter values [5, 6, 73–75]. Some applications, such as coherent perfect absorbers and time-reversal lasers have been elaborated in such settings [76–80], and non-$PT$-symmetric optical potentials with all-real spectra in a coherent atomic system have been realized [71]. So, the studies of the stability of nonlinear modes supported by complex-valued potentials make this topic a part of the very broad field of dynamical stability in various nonlinear systems. A key issue in this field is the study of the best known symmetry breaking instability, that is, the so-called modulation instability (MI) of the corresponding waveforms.

The MI phenomenon was widely investigated in $PT$-symmetric nonlinear Schrödinger (NLS) equation [81–86]. Recently, the MI of constant-amplitude waves has been addressed in models with more general complex potentials by using the plane-wave-expansion method combined with direct numerical simulations, which makes it possible to investigate the bandstructure of the stability problem of spatially periodic solutions in periodic complex-valued potentials [52, 87]. In the present work, we aim to explore the above-mentioned bandstructure for periodic solutions of the NLS equation with periodically modulated third-order dispersion (TOD) and complex potential.

The paper is organized as follows. In the next Section, the model and its reduction are introduced, and the corresponding periodic solutions are presented by solving an inverse problem, which predicts the periodic potential and the TOD supporting the required phase-gradient structure of the corresponding periodic solutions. In Sec. 3, we focus on the analysis of the bandstructure of the linear stability problem of the periodic solutions by employing the plane-wave-expansion method. The
conclusions are given in Sec. 4.

2. THE GOVERNING MODEL AND PERIODIC SOLUTIONS

We consider the NLS equation with periodically-modulated TOD and non-Hermitian potential, which is written in a scaled form, cf. Ref. [52]:

\[
i\partial_t \Psi + \frac{1}{2} \partial_{xx}^2 \Psi + \beta(x) \frac{1}{6} \partial_{xxx} \Psi + V(x)\Psi + g|\Psi|^2 \Psi = 0.
\] (1)

Here \(\Psi(x,z)\) is the slowly varying envelope of the electric field, and \(z\) and \(x\) are the propagation distance and the transverse coordinate, respectively. The function \(\beta(x)\) stands for the TOD, which can appear in a waveguide carrying a photonic-crystal structure to modify the simple paraxial form of the diffraction [88, 89]. In Eq. (1), \(V(x) \equiv V_R(x) + iV_I(x)\) is the complex-valued potential, which can be implemented in optical settings by combining the spatially modulated refractive index and spatially distributed gain/loss elements [10]. The nonlinearity can be either self-focusing \((g > 0)\) or self-defocusing \((g < 0)\), the latter has been proved to be possible in semiconductor materials [90].

We are looking for stationary solutions of Eq. (1) as

\[
\Psi(x,z) = \Phi(x) \exp(i\mu z),
\] (2)

where \(\mu\) is the real propagation constant and the complex field profile \(\Phi(x)\) is determined by the following nonlinear differential equation:

\[
-\mu \Phi + \frac{1}{2} \Phi'' + \frac{i\beta(x)}{6} \Phi''' + V(x)\Phi + g|\Phi|^2 \Phi = 0,
\] (3)

where the prime stands for \(d/dx\). Further, we define the real amplitude and phase

\[
\Phi(x) = H(x) \exp[i\Theta(x)],
\] (4)

for which the complex equation (3) splits into two real equations:

\[
0 = \frac{1}{2} H'' - \mu H - \frac{1}{2} H(\Theta)^2 - \frac{\beta(x)}{2} \Theta' H'' - \frac{\beta(x)}{2} \Theta'' H + \frac{\beta(x)}{6} \Theta''' H + V_R H + g H^3,
\] (5)

\[
0 = H' \Theta' + \frac{1}{2} \Theta'' H + \frac{\beta(x)}{6} H''' - \frac{\beta(x)}{2} (\Theta')^2 H' - \frac{\beta(x)}{2} \Theta' \Theta'' H + V_I H.
\] (6)

Equations (5) and (6) may be addressed as an inverse problem, which provides a required form of the solution, \(H(x)\) and \(\Theta(x)\), by selecting the real and imaginary
parts of a particular complex-valued potential:
\[
V_R(x) = \mu - \frac{1}{2} \frac{H''}{H} + \frac{1}{2} \left( \Theta' \right)^2 + \frac{\beta(x)}{2} \frac{\Theta'' H''}{H} + \frac{\beta(x)}{2} \frac{\Theta' H'}{H} + \frac{\beta(x)}{6} \Theta''' - \frac{\beta(x)}{6} \left( \Theta' \right)^3 - gH^2,
\]
(7)
\[
V_I(x) = -\frac{H' \Theta'}{H} - \frac{1}{2} \Theta'' + \frac{\beta(x)}{2} \frac{H''}{H} + \frac{\beta(x)}{2} \frac{(\Theta')^2 H'}{H} + \frac{\beta(x)}{2} \Theta'' H' + \frac{\beta(x)}{2} \Theta'' H, \quad (8)
\]
This approach that is based on the inverse problem was previously elaborated in various contexts related to NLS-type equations [91–95].

To acquire the basic periodic solution to Eq. (1), avoiding singularities in the complex potential, we set
\[
\frac{H''}{H} = -\omega^2, \quad \Theta'(x) = V_0 H(x), \quad \text{and} \quad \beta(x) = V_1 H(x),
\]
where \(\omega, V_0, \) and \(V_1\) are the real constants. Accordingly, we have \(H(x) = A \cos(\omega x + \phi)\) and \(\Theta(x) = V_0 \int H(x) dx\) with two arbitrary real constants \(A\) and \(\phi\). Thus, the periodic solution of Eq. (1) can be written as
\[
\Psi(x, z) = H(x) e^{i \Xi(x, z)},
\]
(9)
with the corresponding real and imaginary parts of the complex potential
\[
V_R(x) = \mu + \frac{\omega^2}{2} + \left( \frac{V_0^2}{2} - g \right) H^2 + \frac{2V_0 V_1}{3} H H'' + \frac{V_0^2 V_1}{6} (H')^2 - \frac{V_0^3 V_1}{6} H^4,
\]
(10)
\[
V_I(x) = -\frac{3V_0 H'}{2} - \frac{V_1 H''}{6} + \frac{V_0^2 V_1}{6} H^2 H',
\]
(11)
and the periodically modulated TOD
\[
\beta(x) = V_1 H(x),
\]
(12)
where \(H(x) = A \cos(\omega x + \phi)\) and \(\Xi(x, z) = \mu z + (AV_0/\omega) \sin(\omega x + \phi)\).

It should be noted that the periodic solution exists in the linear limit \((g = 0)\), as well as for an arbitrary strength of the nonlinearity \((g \neq 0)\). Also, it can be shown that the function \(H(x)\) determines the power flow from the gain to the loss regions, the respective Poynting vector, \(S = (i/2)(\Psi \partial \Psi^*/\partial x - \Psi^* \partial \Psi /\partial x)\), takes a very simple form, \(S = V_0 H^3\), which is related to the gain/loss parameter \(V_0\).

3. THE BANDSTRUCTURE OF THE STABILITY PROBLEM OF PERIODIC SOLUTIONS

In this Section, we address the necessary details of the stability problem of the periodic solution (9) by employing the plane-wave-expansion method and the results will be verified by direct numerical simulations. Indeed, the familiar linear-stability analysis can be applied to Eq. (1) to analyze the stability of the periodic solution,
but it cannot be used to obtain the stability domains in the presence of a periodic potential.

Firstly, the linear stability analysis can be performed by adding a small perturbation to the periodic solution (9):

\[ \Psi(x, z) = \left[ H + \varepsilon F_\lambda(x) e^{i\lambda z} + \varepsilon G^*_\lambda(x) e^{-i\lambda z} \right] e^{i\Xi(x, z)}, \]

(13)

where “*” stands for the complex conjugation and \( \varepsilon \) is a real infinitesimal amplitude of the perturbation with complex eigenfunctions \( F_\lambda(x) \) and \( G_\lambda(x) \), which are related to the complex eigenvalue \( \lambda \). As usual, the imaginary part of \( \lambda \) determines the instability growth rate of the perturbation. The substitution of the expression (13) into Eq. (1) and subsequent linearization leads to the eigenvalue problem in the matrix form

\[ \begin{pmatrix} L_1 & gH^2 \\ -gH^2 & L_2 \end{pmatrix} \begin{pmatrix} F_\lambda(x) \\ G_\lambda(x) \end{pmatrix} = \lambda \begin{pmatrix} F_\lambda(x) \\ G_\lambda(x) \end{pmatrix}, \]

(14)

where the operators \( L_1 \) and \( L_2 \) are given by

\[
\begin{align*}
L_1 &= iV_0 H \partial_x - iV_0 H_x + \frac{1}{2} \partial_{xx} + \frac{V_1 V_0}{2} HH_{xx} - \frac{iV_1}{6} H_{xxx} \\
&\quad + \frac{iV_1 V_2}{2} H^2 H_x + \frac{iV_1 V_0}{6} H \partial_{xxx} - \frac{V_1 V_0}{2} HH_x \partial_x + gH^2 \\
&\quad - \frac{V_1 V_0}{2} H^2 \partial_{xx} - \frac{iV_0^2 V_1}{2} H^3 \partial_x + \frac{1}{2} \omega^2 + \frac{V_1 V_0}{2} H_x^2,
\end{align*}
\]

(15)

\[
\begin{align*}
L_2 &= iV_0 H \partial_x - iV_0 H_x + \frac{1}{2} \partial_{xx} - \frac{V_1 V_0}{2} HH_{xx} + \frac{iV_1}{6} H_{xxx} \\
&\quad + \frac{iV_1 V_2}{2} H^2 H_x + \frac{iV_1 V_0}{6} H \partial_{xxx} + \frac{V_1 V_0}{2} HH_x \partial_x - gH^2 \\
&\quad + \frac{V_1 V_0}{2} H^2 \partial_{xx} - \frac{iV_0^2 V_1}{2} H^3 \partial_x - \frac{1}{2} \omega^2 - \frac{V_1 V_0}{2} H_x^2.
\end{align*}
\]

(16)

The linear eigenvalue problem (14) can be solved by the finite difference method. It should be emphasized that the result does not include the bandstructure of the stability problem for the periodic solution (9). In the following, we will apply the plane-wave-expansion method based on the formula (14) [52, 87] to study the issue. In the framework of this method in its general form, because \( H(x) \) is a periodic function with the period of \( 2\pi/\omega \), the perturbation eigenmodes \( F_\lambda(x) \) and \( G_\lambda(x) \), along \( H(x) \)
itself, are expanded into Fourier series, according to the Floquet-Bloch theorem:

\[
\begin{bmatrix} F_\lambda(x) \\ G_\lambda(x) \end{bmatrix} = \sum_{n=-\infty}^{+\infty} \begin{bmatrix} u_n(k) \\ v_n(k) \end{bmatrix} e^{i(n\omega+k)x},
\]

\[
H(x) = \sum_{n=-\infty}^{+\infty} H_n e^{in\omega x},
\]

where \(k\) is the Bloch momentum, making the eigenmodes quasiperiodic functions of \(x\). Substituting Eqs. (10), (11), (17), and (18) into the eigenvalue problem (14), one arrives at the following system of linear equations for the perturbation coefficients \(u_n\), \(v_n\) and the eigenvalue \(\lambda(k)\):

\[
\begin{align*}
\lambda u_n &= \alpha_n - 3 u_{n-3} + \beta_n - 2 u_{n-2} + \tau v_{n-2} \\
&\quad + \gamma_{n-1} u_{n-1} + \theta_n v_n + \sigma_n + 1 u_{n+1} \\
&\quad + \eta_{n+2} u_{n+2} + \chi v_{n+2} + \delta_n + 3 u_{n+3}, \\
\lambda v_n &= \alpha_n - 3 v_{n-3} - \tau u_{n-2} - \beta_n - 2 v_{n-2} \\
&\quad + \gamma_{n-1} v_{n-1} - \theta_n u_n + \sigma_n + 1 v_{n+1} \\
&\quad - \chi u_{n+2} - \eta_{n+2} v_{n+2} + \delta_n + 3 v_{n+3}.
\end{align*}
\]

Here we have defined

\[
\alpha_n = \frac{A^3 V_0^2 V_1}{16} [-q + (qn+k)] e^{3i\phi},
\]

\[
\beta_n = \frac{A^2}{8} \left\{ 2g + V_0 V_1 \left[ (qn+k)^2 + q(qn+k) - 2q^2 \right] \right\} e^{2i\phi},
\]

\[
\tau = \frac{g A^2}{4} e^{2i\phi},
\]

\[
\gamma_n = \frac{A}{12} \left[ V_1 (qn+k)^3 - 6V_0(qn+k) + 6V_0q - V_1 q^3 \right] e^{i\phi},
\]

\[
\theta_n = -\frac{1}{2} (qn+k)^2 + \frac{1}{2} q^2,
\]

\[
\sigma_n = \frac{A}{12} \left[ V_1 (qn+k)^3 - 6V_0(qn+k) - 6V_0q + V_1 q^3 \right] e^{-i\phi},
\]

\[
\eta_n = \frac{A^2}{8} \left\{ 2g + V_0 V_1 \left[ (qn+k)^2 - q(qn+k) - 2q^2 \right] \right\} e^{-2i\phi},
\]

\[
\chi = \frac{g A^2}{4} e^{-2i\phi},
\]

\[
\delta_n = \frac{A^3 V_0^2 V_1}{16} [q + (qn+k)] e^{-3i\phi}.
\]

The instability growth rate of the periodic solution is defined as the largest imaginary
part of $\lambda(k)$, in the set of the eigenvalues for a given $k$. Thus, one can acquire the bandstructure of the stability problem for the periodic solution in the periodic potential. As a typical example, Fig. 1 depicts the dependence of $\max[\text{Im} \lambda(k)]$ on $k$ (in the half of the first Brillouin zone) for different values of the TOD parameter $V_1$ in both the linear and nonlinear regimes.

Fig. 1 – (Color online) The dependence of the instability growth rate, i.e., the largest imaginary part of eigenvalues $\lambda(k)$, on the Bloch wavenumber $k$ (in the half of the first Brillouin zone) for different TOD coefficients $V_1$. (a), (b), and (c) are the results for the linear case ($g = 0$), the self-focusing ($g = 0.5$), and the self-defocusing ($g = -0.5$) nonlinearity, respectively, where the black dotted, red dashed, gray solid, blue short-dashed, and green short-dotted curves correspond to $V_1 = -1.0, -0.2, 0, 0.2$, and 1, respectively. Here the other parameters are $A = 0.5, V_0 = 0.1, \omega = 1, \phi = 0$, and $\mu = -0.5$.

In the linear regime, as seen in Fig. 1(a), the stability band, i.e., the interval of wavenumbers $k$ in which the instability growth rate vanishes, is mainly placed in the center of the first Brillouin zone, and its length grows with the increase of $V_1$. The situation changes in the presence of nonlinearity. For the self-focusing nonlinearity,
as shown in Fig. 1(b), for $V_1 = -1$, the eigenvalues with the largest imaginary part are all complex for all $k$ (see the black dotted curve), i.e., there exist no stability band, hence the periodic solutions are linearly unstable to all perturbations. With the increasing of $V_1$, the stability band appears in interval of $[0.28, 0.4]$ for $V_1 = -0.2$, $[0.27, 0.42]$ for $V_1 = 0$, $[0.26, 0.44]$ for $V_1 = 0.2$, and $[0.28, 0.39] \cup (0.39, 0.49]$ for $V_1 = 1$. This means that the periodic solution is stable against perturbations corresponding to the Floquet-Bloch modes within the stability band. For the self-defocusing nonlinearity, as shown in Fig. 1(c), for $V_1 = -1$, the stability band is in the small interval $k \in [0.02, 0.04]$ (see the black dotted curve), and when $V_1$ grows, the situation is similar to the case of self-focusing nonlinearity and the only difference is that the stability band becomes narrower.

Fig. 2 – (Color online) Numerically simulated evolutions of perturbed periodic solutions. (a, b) the linear regime ($g = 0$), (c, d) the self-focusing nonlinear regime ($g = 0.5$), and (e, f) the self-defocusing nonlinear regime ($g = -0.5$). Here (a) $k = 0.15$, $V_1 = 0.2$; (b) $k = 0.48$, $V_1 = 0.2$; (c) $k = 0.15$, $V_1 = 0.2$; (d) $k = 0.3$, $V_1 = 0.2$; (e) $k = 0.15$, $V_1 = -0.2$; (f) $k = 0.38$, $V_1 = -0.2$. The other parameters are the same as in Fig. 1.

To verify the above conclusions, we performed numerical simulations of Eq. (1) by taking inputs in the form of periodic solutions with the addition of small pertur-
bations corresponding to specific Floquet-Bloch eigenmodes. The results are summarized in Fig. 2, for the perturbation amplitude $\varepsilon = 0.01$. Figures 2(a) and 2(b) show the evolutions of the periodic solutions perturbed by the eigenmodes for $k = 0.15$ and $k = 0.48$ as $V_1 = 0.2$ in the linear regime. As predicted by the points “a” and “b” in Fig. 1(a), the periodic solution is stable for $k = 0.15$ and unstable for $k = 0.48$. Figures 2(b) and 2(c) present the evolutions of the periodic solutions perturbed by the eigenmodes for $k = 0.15$ (unstable) and $k = 0.3$ (stable) as $V_1 = 0.2$ in the self-focusing nonlinear regime, while the evolutions for $k = 0.15$ (unstable) and $k = 0.38$ (stable) as $V_1 = -0.2$ in the self-defocusing nonlinear regime are shown in Figs. 2(e) and 2(f). These findings are consistent with the predictions of the linear stability analysis shown in Figs. 1(b) and 1(c) [see the points “c” and “d” in Fig. 1(b) and “e” and “f” in Fig. 1(c)].

Furthermore, the effect of the TOD on the stability band structure of the periodic solution is shown in Fig. 3. In the linear case, as shown in Fig. 3(a), the stability
Fig. 4 – (Color online) The dependence of the instability growth rate, $\max[\text{Im}\lambda(k)]$, on the nonlinear coefficient $g$ and the wavenumber $k$ for (a) $V_1 = 0.2$ and (b) $V_1 = -0.2$. Here, the parameters are the same as in Fig. 1.

Fig. 5 – (Color online) The dependence of the largest imaginary part of the eigenvalues on the amplitude $A$, in the half of the first Brillouin zone for (a) $V_1 = 0.2$, $g = 0$, (b) $V_1 = 0.2$, $g = 0.5$, (c) $V_1 = 0.2$, $g = -0.5$. The other parameters are the same as in Fig. 1.
band is mainly focused on the center of the Brillouin zone, and its length grows as $V_1$ varies from $-1.2$ to $0.69$ and shrinks as $V_1$ varies from $0.89$ to $1.2$. When $0.69 \leq V_1 \leq 0.89$, the eigenvalues are almost real at all $k$, i.e., the periodic solution is stable for all $k$. In the nonlinear regime, the stability problem bandstructure changes significantly. For the self-focusing nonlinearity, as shown in Fig. 3(b), the stability band does not exist when $V_1$ belongs to the interval of $-1.2 \leq V_1 < -0.7$. When $V_1 > -0.7$, the stability band appears and its length grows as $V_1$ varies from $-0.7$ to $0.77$ and shrinks as $V_1$ varies from $0.77$ to $1.2$. For $0.77 \leq V_1 \leq 1.0$, the stability band is extended to the edge of the Brillouin zone. Figure 3(c) presents the case of self-defocusing nonlinearity. It is similar in result to the self-focusing case except the stability band also appears in small ranges of $k$ near the center of the first Brillouin zone.

Next, we will discuss the influence of the nonlinear coefficient $g$ on the bandstructure of the stability problem. Figure 4 presents the dependence of the instability growth rate on the nonlinear coefficient $g$ in the half of the first Brillouin zone for different values of TOD. From it, one can see that the length of the stability band has a maximum at $g = 0$ and decreases with the increasing of $|g|$ up to $g > 1.03$ and $g < -1.10$ for $V_1 = 0.2$, and $g > 0.92$ and $g < -0.71$ for $V_1 = -0.2$. So, the stability domains are narrowing with the increasing of the nonlinear effect.

The above analysis was reported for the stability problem of the periodic solution with the amplitude $A = 0.5$. Now, we are considering the effect of the variation of the amplitude $A$ on the stability intervals. As a typical example, we only discuss the case of $V_1 = 0.2$. Figure 5 presents the dependence of the instability growth rate on the amplitude $A$ in the half of the first Brillouin zone for both linear and nonlinear regimes. One can see that the eigenvalues are almost real at all $k$ for small amplitudes, and with the increasing of $A$, the stability band becomes gradually narrow up to $A = 3.36$ ($g = 0$), $A = 0.69$ ($g = 0.5$), and $A = 0.66$ ($g = -0.5$), beyond which the eigenvalues with the largest imaginary parts are complex at all $k$. Thus, one can conclude that the stability of the periodic solution will be weakened when the amplitude of periodic solution increases.

### 4. CONCLUSIONS

In summary, we have considered a class of periodic solutions of the NLS equation with periodically-modulated TOD and complex-valued potential. The model was built as a solution of the associated inverse problem, which predicts the complex potential and the TOD needed to support periodic nonlinear states with the required phase-gradient structure. The physical setting can be realized in nonlinear optical waveguides, with the appropriate spatial distributions of local gain/loss and TOD.
The analysis was mainly focused on the stability problem of periodic solutions in the associated periodic potential, by means of the plane-wave-expansion method and direct numerical simulations. The obtained results have shown that the stability of the periodic solution will be weakened when either the solution’s amplitude or the nonlinear effect increases. The method elaborated in the present work can be used to analyze the bandstructure of the stability problem for periodic solutions in complex-valued potentials for other types of nonlinear wave equations.

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