APPLICATION OF A TRANSFER MATRIX METHOD TO HOLLOW-CORE BRAGG FIBER WITH A GOLD LAYER

V.A. POPESCU

Politehnica University of Bucharest, Department of Physics 1, 313 Splaiul Independentei, 060042 Bucharest, Romania
E-mail: vapopescu@yahoo.com

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Abstract. For a hollow-core Bragg fiber, the field is represented by a Bessel function of the first kind in the core region, a linear combination of Bessel functions of the first and second kinds in the dielectric interior layers, a linear combination of the Hankel functions in the gold region and a Hankel function of the first kind in the external infinite medium. Our analytical method is applied for different structures made from 19, 11, and 5 layers. When a high index material just before the outermost region of a hollow-core Bragg fiber is replaced by a gold layer, the optical confinement for the TE$_{01}$ mode in the core is increased about ten times. If the gold layer is located between the first and the penultimate layer, the loss for the same mode is increased.

Key words: sensors, hollow-core fibers, surface plasmon resonance, finite element method.

1. INTRODUCTION

The transfer matrix method has been used over the past decades for the analysis of planar waveguides [1], optical fibers [2–5], fiber gratings [6–7], and fiber based plasmonic sensors [8–11].

The solutions of the wave equation with cylindrical symmetry for the electric field $\vec{E}$ and the magnetic field $\vec{H}$ in the fiber yield to a dependency in the form $e^{-i\beta z}$ in the $z$ direction and $e^{i\alpha\phi}$ around the circumference of the fiber, and to the radial solutions $\Psi$ and $\Phi$ [3]:

$$\vec{E} = \left[ \frac{1}{r} \frac{\partial \Psi}{\partial \phi} - \frac{\beta}{\omega \varepsilon_0 n^2} \frac{\partial \Phi}{\partial r} \right] \vec{r} - \left[ \frac{\partial \Psi}{\partial r} + \frac{\beta}{\omega \varepsilon_0 n^2} \frac{\partial \Phi}{\partial \phi} \right] \vec{\phi},$$

and
\[ \vec{H} = \left[ \frac{1}{r} \frac{\partial \Phi}{\partial \phi} + \frac{\beta}{\omega \mu_0} \frac{\partial \Psi}{\partial r} \right] \hat{\imath}_r + \left[ -\frac{\partial \Phi}{\partial r} + \frac{\beta}{\omega \mu_0 r} \frac{\partial \Psi}{\partial \phi} \right] \hat{\imath}_\phi \\
+ \left( \frac{k^2 n^2 - \beta^2}{i \omega \mu_0} \right) \hat{\imath}_z e^{-i \beta z}, \quad (2) \]

where \( \mu_0 \) is the free space magnetic permeability, \( \varepsilon_0 \) is the vacuum permittivity, \( \omega \) is the angular frequency, \( \hat{\imath}_r, \hat{\imath}_\phi, \) and \( \hat{\imath}_z \) are the unit radial, tangential, and axial vectors and the mode index \( \nu \) must be an integer to ensure periodic solutions with period \( 2\pi \). For a hollow-core Bragg fiber, the radial solutions \( \Psi \) and \( \Phi \) are represented by a Bessel function of the first kind \( J_n \) in the core region, a linear combination of Bessel functions of the first and second kinds \( J_n \) and \( Y_n \) in the dielectric interior layers, a linear combination of the Hankel functions \( H_n^1 \) and \( H_n^2 \) in the gold region, and a Hankel function of the first kind \( H_n^1 \) in the external infinite medium. The continuity conditions require that the tangential components \( \phi \) and \( z \) of the electric field \( E \) and the magnetic field \( H \) must be matched at the different layer interfaces.

A low loss of the \( \text{TE}_{01} \) mode in a hollow-core Bragg fiber with large radius was obtained by using a high contrast for the refractive indices \( n_1 = 1, n_2 = 4.6, n_3 = 1.6, n_{N-1} = 4.6, n_N = 1.6 \) of the alternating layers [12], where \( N \) is the number of the layers.

In the proposed device, the high index material just before the outermost region of a hollow-core Bragg fiber is replaced by a gold layer and the optical confinement in the core is increased about ten times.

2. ANALYTICAL METHOD FOR A HOLLOW-CORE BRAGG FIBER WITH A GOLD LAYER

For a hollow-core Bragg fiber with five layers, we have [5, 8]:

\[
M_0 \begin{bmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{bmatrix} = M_1 \begin{bmatrix} A_2 \\ B_2 \\ C_2 \\ D_2 \end{bmatrix} = M_2 \begin{bmatrix} A_3 \\ B_3 \\ C_3 \\ D_3 \end{bmatrix}; \quad (3)
\]

\[
M_4 \begin{bmatrix} A_3 \\ B_3 \\ C_3 \\ D_3 \end{bmatrix} = M_5 \begin{bmatrix} A_4 \\ B_4 \\ C_4 \\ D_4 \end{bmatrix} = M_6 \begin{bmatrix} A_5 \\ B_5 \\ C_5 \\ D_5 \end{bmatrix}; \quad (4)
\]
Application of a transfer matrix method to hollow-core Bragg fiber with a gold layer

\[ M_4 \begin{pmatrix} A_3 \\ B_3 \\ C_3 \\ D_3 \end{pmatrix} = M_{5g} \begin{pmatrix} A_4 \\ B_4 \\ C_4 \\ D_4 \end{pmatrix}, \quad M_{6g} \begin{pmatrix} A_4 \\ B_4 \\ C_4 \\ D_4 \end{pmatrix} = M_f \begin{pmatrix} A_5 \\ B_5 \\ C_5 \\ D_5 \end{pmatrix}, \]

where the index \( g \) refers to a gold layer and the elements of the matrices \( M_0, M_1, M_2, M_3, M_4, M_{5g}, M_{6g}, \) and \( M_f \) are

\[ M_{01}^{11} = -\frac{u_1^2}{n_1^2} J_v(u_1 \eta_1), \quad M_{01}^{12} = 0, \quad M_{01}^{13} = 0, \quad M_{01}^{14} = 0, \]

\[ M_{01}^{21} = -J_v'(u_1 \eta_1), \quad M_{01}^{22} = 0, \quad M_{01}^{23} = \frac{i \nu \beta}{\omega \mu_0 \eta_1} J_v(u_1 \eta_1), \quad M_{01}^{24} = 0, \]

\[ M_{01}^{31} = 0, \quad M_{01}^{32} = 0, \quad M_{01}^{33} = u_1^2 J_v(u_1 \eta_1), \quad M_{01}^{34} = 0, \]

\[ M_{01}^{41} = -\frac{i \nu \beta}{\omega \epsilon \mu_0 \eta_1} J_v(u_1 \eta_1), \quad M_{01}^{42} = 0, \quad M_{01}^{43} = -J_v'(u_1 \eta_1), \quad M_{01}^{44} = 0, \]

\[ M_{11}^{11} = -\frac{u_2^2}{n_2^2} J_v(u_2 \eta_1), \quad M_{11}^{12} = -\frac{u_2^2}{n_2^2} Y_v(u_2 \eta_1), \quad M_{11}^{13} = 0, \quad M_{11}^{14} = 0, \]

\[ M_{11}^{21} = -J_v'(u_2 \eta_1), \quad M_{11}^{22} = -Y_v'(u_2 \eta_1), \]

\[ M_{11}^{23} = \frac{i \nu \beta}{\omega \mu_0 \eta_1} J_v(u_2 \eta_1), \quad M_{11}^{24} = \frac{i \nu \beta}{\omega \mu_0 \eta_1} Y_v(u_2 \eta_1), \]

\[ M_{11}^{31} = 0, \quad M_{11}^{32} = 0, \quad M_{11}^{33} = u_2^2 J_v(u_2 \eta_1), \quad M_{11}^{34} = u_2^2 Y_v(u_2 \eta_1), \]
\begin{align*}
M_1^{41} &= -\frac{i\nu \beta}{\omega \varepsilon_0 n_2^2 \eta} J_{\nu}(u_2 \eta), \quad M_1^{42} = -\frac{i\nu \beta}{\omega \varepsilon_0 n_2^2 \eta} Y_{\nu}(u_2 \eta), \\
M_1^{43} &= -J'_{\nu}(u_2 \eta), \\
M_1^{44} &= -Y'_{\nu}(u_2 \eta), \\
M_2 &= M_1(\eta \rightarrow r_2), \\
M_3 &= M_1(\eta \rightarrow r_2, u_2 \rightarrow u_3, n_2 \rightarrow n_3), \\
M_4 &= M_1(\eta \rightarrow r_3, u_2 \rightarrow u_3, n_2 \rightarrow n_3), \\
M_5 &= M_1(\eta \rightarrow r_3, u_2 \rightarrow u_4, n_2 \rightarrow n_4), \\
M_{5,g} &= M_1(\eta \rightarrow r_3, u_2 \rightarrow u_4, n_2 \rightarrow n_4, J_{\nu} \rightarrow H_{1\nu}, Y_{\nu} \rightarrow H_{2\nu}), \\
M_6 &= M_1(\eta \rightarrow r_4, u_2 \rightarrow u_4, n_2 \rightarrow n_4), \\
M_{6,g} &= M_1(\eta \rightarrow r_4, u_2 \rightarrow u_4, n_2 \rightarrow n_4, J_{\nu} \rightarrow H_{1\nu}, Y_{\nu} \rightarrow H_{2\nu}), \\
M_{11} &= 0, M_{12} = -\frac{w_5^2}{n_5^2} H_{1\nu}(w_5 r_4), M_{13} = 0, M_{14} = 0.
\end{align*}
\[ M_{f}^{21} = 0, M_{f}^{22} = -H_{1\nu} (w_{5} r_{4}), M_{f}^{23} = 0, \]
\[ M_{f}^{24} = \frac{i \nu \beta}{\omega \mu_{0} r_{4}} H_{1\nu} (w_{5} r_{4}), \] (24)
\[ M_{f}^{31} = 0, M_{f}^{32} = 0, M_{f}^{33} = 0, M_{f}^{34} = w_{4}^{2} K_{\nu} (w_{4} r_{3}), \] (25)
\[ M_{f}^{41} = 0, M_{f}^{42} = -\frac{i \nu \beta}{\omega \varepsilon_{0} n_{5}^{2} r_{4}} H_{1\nu} (w_{5} r_{4}), M_{f}^{43} = 0, \]
\[ M_{f}^{44} = -H_{1\nu} (w_{5} r_{4}), \] (26)
where
\[ u_{1} = \sqrt{(kn_{1})^{2} - \beta^{2}}, \quad u_{2} = \sqrt{(kn_{2})^{2} - \beta^{2}}, \quad u_{3} = \sqrt{(kn_{3})^{2} - \beta^{2}}, \] (27)
\[ u_{4} = \sqrt{(kn_{4})^{2} - \beta^{2}}, \quad w_{5} = \sqrt{(kn_{5})^{2} - \beta^{2}}, \] (28)
and where prime represents the differentiation with respect to the radial variable r:
\[ F_{\nu}'(u_{i} r_{j}) = \frac{u_{i}}{2} \left[ F_{\nu-1}(u_{i} r_{j}) - F_{\nu+1}(u_{i} r_{j}) \right], F = J, Y, H_{1}, H_{2}. \] (29)

The complex propagation constant \( \beta = \beta_{r} + i \beta_{i} \) at a modal index \( \nu \) is determined from the dispersion equation – the determinant \( (\Delta) \) formed by the coefficients \( A_{1}, C_{1}, B_{4}, \) and \( D_{4} \) of the equations
\[- \frac{u_{i}^{2}}{n_{1}^{2}} J_{\nu}(u_{i} r_{1}) A_{1} - B_{12} B_{5} - B_{14} D_{5} = 0, \] (30)
\[- J_{\nu}(u_{i} r_{1}) A_{1} + \frac{i \nu \beta}{\omega \mu_{0} n_{1}} J_{\nu}(u_{i} r_{1}) C_{1} - B_{22} B_{5} - B_{24} D_{5} = 0, \] (31)
\[ u_{1}^{2} J_{\nu}(u_{i} r_{1}) C_{1} - B_{32} B_{5} - B_{34} D_{5} = 0, \] (32)
\[- \frac{\nu B}{\omega \varepsilon_0 n_1^2 \eta_1} J_\nu (u_1 \eta_1) A_1 - J'_\nu (u_1 \eta_1) C_1 - B_{42v} B_5 - B_{44v} D_5 = 0, \quad (33)\]

must vanish:

\[\Delta = 0, \quad (34)\]

where

\[\Delta = \begin{vmatrix}
- \frac{u_1^2}{n_1^2} J_\nu (u_1 \eta_1) & 0 & -B_{12v} & -B_{14v} \\
- J'_\nu (u_1 \eta_1) & \frac{\nu B}{\omega \mu_0 \eta_1} J_\nu (u_1 \eta_1) & -B_{22v} & -B_{24v} \\
0 & u_1^2 J_\nu (u_1 \eta_1) & -B_{32v} & -B_{34v} \\
- \frac{\nu B}{\omega \varepsilon_0 n_1^2 \eta_1} J_\nu (u_1 \eta_1) & - J'_\nu (u_1 \eta_1) & -B_{42v} & -B_{44v}
\end{vmatrix}, \quad (35)\]

\[M_1 M_2^{-1} M_3 M_4^{-1} M_5 M_6^{-1} M_f = B_v, \quad (36)\]

\[B_v = \begin{pmatrix} 0 & B_{12v} & 0 & B_{14v} \\ 0 & B_{22v} & 0 & B_{24v} \\ 0 & B_{32v} & 0 & B_{34v} \\ 0 & B_{42v} & 0 & B_{44v} \end{pmatrix}. \quad (37)\]

In general, for a fiber with \(N\) layers the matrix elements for the layer just before the outermost region are

\[M_{2(N-2)-1} = M_1 (r_1 \rightarrow r_{N-2}, u_2 \rightarrow u_{N-1}, n_2 \rightarrow n_{N-1}), \quad (38)\]

\[M_{2(N-2)-1,g} = M_1 (r_1 \rightarrow r_{N-2}, u_2 \rightarrow u_{N-1}, n_2 \rightarrow n_{N-1}, J_\nu \rightarrow H_{1v}, Y_v \rightarrow H_{2v}), \quad (39)\]
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$M_{2(N-2)} = M_1(r_1 \to r_{N-1}, u_2 \to u_{N-1}, n_2 \to n_{N-1})$,

$M_{2(N-2),g} = M_1(r_1 \to r_{N-1}, u_2 \to u_{N-1}, n_2 \to n_{N-1}, J_v \to H_{1v}, Y_v \to H_{2v})$.

3. NUMERICAL RESULTS AND DISCUSSION

The analytical method is demonstrated in a hollow-core Bragg fiber, which is made by air in the center of the structure, surrounded by periodic reflector layers with large refractive-index contrast in the cladding. In addition, the high index material just before the outermost region of the fiber is replaced by a gold layer. Thus, the loss for the TE$_{01}$ mode is decreased about ten times.

The optical properties of a Bragg fiber with a hollow-core of large radius and a large refractive-index contrast in periodic layers of the cladding, but without a gold layer were calculated in [12]. The fiber parameters were $r = 13.02 \mu m$, $n_1 = 1$, $d_1 = 0.09444 \mu m$, $n_2 = 4.6$, $d_2 = 0.33956 \mu m$, $n_3 = 1.6$, $\lambda = 1.55 \mu m$, and $N = 19$, where $r$ is the radius of the core, $n_1$ is the refractive index of the air in the core, $d_1$ is the thickness of the layer with high refractive index $n_2$, $d_2$ is the thickness of the layer with low refractive index $n_3$, $\lambda$ is the wavelength and $N$ is the number of the layers.

By using the usual quarter wave condition:

$$d_1 = \frac{\lambda}{4\sqrt{n_2^2 - n_1^2}}$$

$$d_2 = \frac{\lambda}{4\sqrt{n_2^2 - n_1^2}}$$

we have calculated the thickness $d_1 = 0.086303 \mu m$ for the layer with high refractive index ($n_2 = 4.6$) and the thickness $d_2 = 0.310248 \mu m$ for the layer with a low refractive index ($n_3 = 1.6$) in the cladding structure, where $n_1 = 1$ is the refractive index in the core region (air). For the hollow-core Bragg fiber with $N$ layers, $r_1 = 13.02 \mu m$, $n_1 = 1$, $n_2 = n_4 = \ldots = n_{N-1} = 4.6$, $n_3 = n_5 = \ldots = n_N = 1.6$. The refractive index of the gold layer is calculated by the Drude model [13].

Table 1 shows the values of the effective index $\beta/k$, loss $\alpha$, propagation length $L$, and wavelength $\lambda$ for a hollow-core Bragg fiber with $N = 5, 11,$ and 19 layers, $r_1 = 13.02 \mu m$, $n_1 = 1$, $d_1 = 0.086303 \mu m$, $d_2 = 0.310248 \mu m$, $n_2 = n_4 = \ldots = n_{N-1} = 4.6$, $n_3 = n_5 = \ldots = n_N = 1.6$. The symbol * refers to $d_1 = 0.086292 \mu m$ and
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$d_2 = 0.309726 \mu m$ computed with the quarter-wave stack condition in the case of infinite cladding pairs \[14, 15\]:

\[
\frac{\lambda}{d_1} = 2 \sqrt{4(n_2^2 - n_1^2) + \left(\frac{\lambda J_{1,1}}{\pi n_1}\right)^2}, \quad \frac{\lambda}{d_2} = 2 \sqrt{4(n_3^2 - n_1^2) + \left(\frac{\lambda J_{1,1}}{\pi n_1}\right)^2},
\] (43)

where $J_{1,1} = 3.83170597$ is the first root of the Bessel function $J_1$. If we approximate

\[
4(n_2^2 - n_1^2) + \left(\frac{\lambda J_{1,1}}{\pi n_1}\right)^2 \approx 4(n_2^2 - n_1^2)
\] (44)

one obtains the usual quarter wave condition (42). The minimum-loss wavelength $\lambda_{\text{min}}$ shifts ($\Delta \lambda = 20.8 \text{ nm for } N = 19, \Delta \lambda = 40.4 \text{ nm for } N = 11,$ and $\Delta \lambda = 128 \text{ nm for } N = 5$) toward a short wavelength as the number of the layers $N$ becomes small.

**Table 1**

<table>
<thead>
<tr>
<th>$N$</th>
<th>$\beta / k$ (without gold)</th>
<th>$\alpha$ [dB/cm]</th>
<th>$L$ [\mu m]</th>
<th>$\lambda$ [\mu m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.997361 + 4.820915×10^{-7} i</td>
<td>1.697430×10^{-4}</td>
<td>2.558541×10^{10}</td>
<td>1.5500</td>
</tr>
<tr>
<td>5</td>
<td>0.997782 + 4.036744×10^{-7} i</td>
<td>1.549047×10^{-4}</td>
<td>2.803623×10^{10}</td>
<td>1.4222</td>
</tr>
<tr>
<td>11</td>
<td>0.997361 + 2.253839×10^{-10} i</td>
<td>7.935707×10^{-10}</td>
<td>5.472667×10^{10}</td>
<td>1.5500</td>
</tr>
<tr>
<td>11</td>
<td>0.997498 + 2.137121×10^{-10} i</td>
<td>7.726120×10^{-10}</td>
<td>5.621120×10^{13}</td>
<td>1.5096</td>
</tr>
<tr>
<td>19</td>
<td>0.997361 + 8.178485×10^{-10} i</td>
<td>2.879621×10^{-10}</td>
<td>1.508165×10^{10}</td>
<td>1.5500</td>
</tr>
<tr>
<td>19</td>
<td>0.997361 + 8.178337×10^{-10} i</td>
<td>2.879569×10^{-10}</td>
<td>1.508193×10^{10}</td>
<td>1.5500*</td>
</tr>
<tr>
<td>19</td>
<td>0.997432 + 7.962187×10^{-10} i</td>
<td>2.841596×10^{-10}</td>
<td>1.528347×10^{10}</td>
<td>1.5292</td>
</tr>
<tr>
<td>19</td>
<td>0.997432 + 7.948044×10^{-10} i</td>
<td>2.836548×10^{-10}</td>
<td>1.531067×10^{10}</td>
<td>1.5292*</td>
</tr>
</tbody>
</table>

Figure 1 shows the refractive index *versus* the radius of the layers for an air-core Bragg fiber with $N = 5$ when the layer just before the outermost region is a dielectric ($n_4 = 4.6$) or a gold material. Figure 2 shows a quarter of a cross section of the same fiber and a contour plot of the z-component $S_z(x, y)$ of the Poynting vector at the wavelength ($\lambda = 1.4222 \mu m$ when $n_4 = 4.6$ and $\lambda = 1.4188 \mu m$ when $n_4 = 0.503628 - 8.838158i$) of the fiber lowest loss for the TE$_{01}$ mode. Figure 3 shows the loss spectra for the TE$_{01}$ mode for the same fiber without a gold layer.
and with a gold layer. Figure 4 is similar to Fig. 1 but for \( N = 19 \). Figure 5 shows the real part of the effective index \textit{versus} wavelength for the TE\(_{01}\) mode for a fiber with \( N = 19 \). Figure 6 is similar to Fig. 3 but for \( N = 19 \).

Fig. 1 – The refractive index \textit{versus} the radius of the core and cladding layers when \( n_4 = 4.6 \) (a) and \( n_4 = 0.503628 - 8.838158i \) (b) for an air-core Bragg fiber with \( N = 5 \).

Fig. 2 – A quarter of a cross section of a hollow-core Bragg fiber with five layers and a contour plot of the \( z \)-component \( S_z(x, y) \) of the Poynting vector at the wavelength \( \lambda = 1.4222 \mu \text{m} \) when \( n_4 = 4.6 \) (a) and \( \lambda = 1.4188 \mu \text{m} \) when \( n_4 = 0.503628 - 8.838158i \) (b) of the fiber lowest loss for the TE\(_{01}\) mode. The main electric field \( E \) of the TE\(_{01}\) mode is parallel with the surface of the Bragg layers.
Fig. 3 – The loss spectra for the TE$_{01}$ mode in an air-core Bragg fiber with $N = 5$, without a gold layer (a) and with a gold layer (b).

Fig. 4 – The refractive index versus the radius of the core and cladding layers when $n_{18} = 4.6$ (a) and $n_{18} = 0.503628 - 8.838158i$ (b) for an air-core Bragg fiber with $N = 19$. 
Fig. 5 – The real part of the effective index versus the wavelength for the TE\textsubscript{01} mode for an air-core Bragg fiber with $N = 19$, without a gold (a) and with (b) a single gold (for $n_{19}$) layer.

Fig. 6 – The loss spectra for the TE\textsubscript{01} mode in an air-core Bragg fiber with $N = 19$, without a gold layer (a) and with a gold layer (b).
Table 2 shows the values of β/κ, α, L and λ for a hollow-core Bragg fiber with N = 5, 11, and 19 layers, r₁ = 13.02 μm, n₁ = 1, d₁ = 0.086303 μm, d₂ = 0.310248 μm, n₂ = n₄ =...= nₙ₋₃ = 4.6, nₙ₋₁ = nₙ, nₙ₊₁ = nₙ₊₂ =...= nₙ = 1.6, when a high index material just before the outermost region is replaced by a gold layer.

The symbol * refers to d₁ = 0.086292 μm and d₂ = 0.309726 μm computed with the quarter-wave stack condition in the case of infinite cladding pairs. The minimum-loss wavelength λₖ/min shifts (Δλ = 5.7 nm for N = 19, Δλ = 13.6 nm for N = 11, and Δλ = 131.2 nm for N = 5) toward a short wavelength as the number of the layers N becomes small.

<table>
<thead>
<tr>
<th>N</th>
<th>β/κ (with gold)</th>
<th>α [dB/cm]</th>
<th>L [μm]</th>
<th>λ [μm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.997362 – 4.731900 × 10⁻⁶ i</td>
<td>1.666089 × 10⁻⁶</td>
<td>2.606671 × 10⁻⁶</td>
<td>1.5500</td>
</tr>
<tr>
<td>5</td>
<td>0.997795 – 4.205448 × 10⁻⁶ i</td>
<td>1.617653 × 10⁻⁶</td>
<td>2.684720 × 10⁻⁶</td>
<td>1.4188</td>
</tr>
<tr>
<td>11</td>
<td>0.997361 – 2.213349 × 10⁻⁶ i</td>
<td>7.793137 × 10⁻⁶</td>
<td>5.72781 × 10⁻⁶</td>
<td>1.5500</td>
</tr>
<tr>
<td>11</td>
<td>0.997408 – 2.188955 × 10⁻⁶ i</td>
<td>7.775472 × 10⁻⁶</td>
<td>5.85442 × 10⁻⁶</td>
<td>1.5364</td>
</tr>
<tr>
<td>19</td>
<td>0.997361 – 8.034501 × 10⁻⁶ i</td>
<td>2.828925 × 10⁻⁶</td>
<td>1.535193 × 10⁻⁶</td>
<td>1.5500</td>
</tr>
<tr>
<td>19</td>
<td>0.997361 – 8.025363 × 10⁻⁶ i</td>
<td>2.825707 × 10⁻⁶</td>
<td>1.536941 × 10⁻⁶</td>
<td>1.5500*</td>
</tr>
<tr>
<td>19</td>
<td>0.997381 – 7.995152 × 10⁻⁶ i</td>
<td>2.825460 × 10⁻⁶</td>
<td>1.537075 × 10⁻⁶</td>
<td>1.5443</td>
</tr>
<tr>
<td>19</td>
<td>0.997381 – 7.985089 × 10⁻⁶ i</td>
<td>2.821904 × 10⁻⁶</td>
<td>1.539012 × 10⁻⁶</td>
<td>1.5443</td>
</tr>
</tbody>
</table>

Table 3 shows the values of the loss α and amplitude sensitivity Sₐ for the TE₀₁ mode for two values of the refractive index of the first (interior) and the last (exterior) layer for a hollow-core Bragg fiber with N = 19 layers, r₁ = 13.02 μm, n₁ = 1, d₁ = 0.086303 μm, d₂ = 0.310248 μm with and without gold.

Table 3

<table>
<thead>
<tr>
<th>N</th>
<th>n₁</th>
<th>n₁₈</th>
<th>n₁₉</th>
<th>α [dB/cm]</th>
<th>Sₐ [RIU⁻¹]</th>
<th>λ [μm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>1</td>
<td>4.6</td>
<td>1.6</td>
<td>2.841596 × 10⁻⁶</td>
<td>1.5292</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>1</td>
<td>4.6</td>
<td>1.6</td>
<td>2.844500 × 10⁻⁶</td>
<td>1.022</td>
<td>1.5292</td>
</tr>
<tr>
<td>19</td>
<td>1</td>
<td>4.6</td>
<td>1.6</td>
<td>2.808443 × 10⁻⁶</td>
<td>-11.624</td>
<td>1.5291</td>
</tr>
<tr>
<td>19</td>
<td>1</td>
<td>0.572174–9.621939i</td>
<td>1.6</td>
<td>2.825460 × 10⁻⁶</td>
<td>1.5443</td>
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<td>1</td>
<td>0.572174–9.621939i</td>
<td>1.6</td>
<td>2.825433 × 10⁻⁶</td>
<td>-0.00971</td>
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<tr>
<td>19</td>
<td>1</td>
<td>0.572174–9.621939i</td>
<td>1.6</td>
<td>2.794629 × 10⁻⁶</td>
<td>-10.912</td>
<td>1.5443</td>
</tr>
</tbody>
</table>
In the case without gold, the differences between the losses are:

\[ \alpha(n_1 = 1) - \alpha(n_1 = 1.001) = 3.3153 \times 10^{-11} \text{ dB/cm} \] and

\[ \alpha(n_{10} = 1.6) - \alpha(n_{10} = 1.601) = -2.904 \times 10^{-12} \text{ dB/cm}. \]

In the case with gold, the differences between the losses are:

\[ \alpha(n_1 = 1) - \alpha(n_1 = 1.001) = 3.0831 \times 10^{-12} \text{ dB/cm} \] and

\[ \alpha(n_{10} = 1.6) - \alpha(n_{10} = 1.601) = 2.7 \times 10^{-15} \text{ dB/cm}. \]

Figure 7 shows the loss spectra for two values of the refractive index of the core layer \((n_a = 1 \text{ and } n_a = 1.001)\) and the amplitude sensitivity for the TE\(_{01}\) mode for an air-core Bragg fiber with \(N = 19\), without a gold layer. In this case the maximum of the amplitude sensitivity is \(S_A = -11.624 \text{ RIU}^{-1}\) at \(\lambda = 1.5291 \mu\text{m}\).

Figure 8 is similar to Fig. 7 but for the case with a gold layer. The maximum of the amplitude sensitivity is \(S_A = -10.912 \text{ RIU}^{-1}\) at \(\lambda = 1.5443 \mu\text{m}\).

Figure 9 is similar to Fig. 7 but for two values of the refractive index of the exterior layer \((n_a = 1.6 \text{ and } n_a = 1.601)\). The maximum of the amplitude sensitivity is only \(S_A = 1.022 \text{ RIU}^{-1}\) at \(\lambda = 1.5292 \mu\text{m}\).

Figure 10 is similar to Fig. 9 but for the case with a gold layer. In this case the maximum of the amplitude sensitivity is smaller \((S_A = -0.00971 \text{ RIU}^{-1}\) at \(\lambda = 1.5443 \mu\text{m})\).

Figure 11 shows the loss \textit{versus} the thickness \(t_g\) of the gold layer in \(n_{\text{layer}} = 5\) for the TE\(_{01}\) mode in an air-core Bragg fiber with \(N = 6\). One observe that optical confinement for the TE\(_{01}\) mode in the core is increased (the loss is decreased) with the thickness of the gold layer.
Fig. 8 – The loss spectra (a) for two values of the analyte (interior) refractive index ($n_a = 1$ and $n_a = 1.001$) and the amplitude sensitivity (b) for the TE$_{01}$ mode for an air-core Bragg fiber with $N = 19$, with a single gold layer.

Fig. 9 – The loss spectra (a) for two values of the analyte (exterior) refractive index ($n_a = 1.6$ and $n_a = 1.601$) and the amplitude sensitivity (b) for the TE$_{01}$ mode for an air-core Bragg fiber with $N = 19$, without a gold layer.
Fig. 10 – The loss spectra (a) for two values of the analyte (exterior) refractive index ($n_a = 1.6$ and $n_a = 1.601$) and the amplitude sensitivity (b) for the TE_{01} mode for an air-core Bragg fiber with $N = 19$, with a single gold layer ($N = 18$).

Fig. 11 – The loss versus the thickness $t_g$ of the gold layer in $N = 5$ for the TE_{01} mode in an air-core Bragg fiber with $N = 6$. 
Figure 12 shows the loss in the logarithmic scale versus the number \( n_{\text{layer}} \) of the gold layer and the refractive index versus the radius of the layers when \( n_{\text{layer}} = 10 \).

If the gold layer is located between the first and the penultimate layer, the loss for the same mode is increased. Table 4 shows the values of the effective index \( \beta/k \), loss \( \alpha \), and propagation length \( L \) for an air-core Bragg fiber with \( N = 19 \) layers, \( r_1 = 13.02 \, \mu m \), \( n_1 = 1 \), \( d_1 = 0.086303 \, \mu m \), \( d_2 = 0.310248 \, \mu m \), \( \lambda = 1.55 \, \mu m \), when the gold layer is located between the first and the last layer.

<table>
<thead>
<tr>
<th>( n_{\text{layer}} )</th>
<th>( \beta/k ) (with gold)</th>
<th>( \alpha ) [dB/cm]</th>
<th>( L ) [( \mu m )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.997371 – 6.114745 \times 10^{-4} , i</td>
<td>2.152984 \times 10^{4}</td>
<td>2.017174 \times 10^{5}</td>
</tr>
<tr>
<td>4</td>
<td>0.997762 – 4.774691 \times 10^{-5} , i</td>
<td>1.681155 \times 10^{5}</td>
<td>2.583310 \times 10^{6}</td>
</tr>
<tr>
<td>6</td>
<td>0.997361 – 3.707468 \times 10^{-6} , i</td>
<td>1.305389 \times 10^{7}</td>
<td>3.326936 \times 10^{7}</td>
</tr>
<tr>
<td>8</td>
<td>0.997361 – 2.877559 \times 10^{-8} , i</td>
<td>1.013180 \times 10^{8}</td>
<td>4.286449 \times 10^{8}</td>
</tr>
<tr>
<td>10</td>
<td>0.997361 – 2.233364 \times 10^{-11} , i</td>
<td>7.863610 \times 10^{10}</td>
<td>5.522539 \times 10^{10}</td>
</tr>
<tr>
<td>12</td>
<td>0.997361 – 1.733384 \times 10^{-14} , i</td>
<td>6.103194 \times 10^{12}</td>
<td>7.118555 \times 10^{12}</td>
</tr>
<tr>
<td>14</td>
<td>0.997361 – 1.345284 \times 10^{-17} , i</td>
<td>4.736706 \times 10^{14}</td>
<td>9.168703 \times 10^{14}</td>
</tr>
<tr>
<td>16</td>
<td>0.997361 – 1.043499 \times 10^{-19} , i</td>
<td>3.674130 \times 10^{16}</td>
<td>1.182033 \times 10^{16}</td>
</tr>
<tr>
<td>18</td>
<td>0.997361 – 8.034501 \times 10^{-21} , i</td>
<td>2.828925 \times 10^{18}</td>
<td>1.535193 \times 10^{18}</td>
</tr>
</tbody>
</table>
4. CONCLUSION

When a high index material just before the outermost region of a hollow-core Bragg fiber is replaced by a gold layer, the optical confinement for the TE\textsubscript{01} mode in the core is increased about ten times for any number of layers. A very good optical confinement is obtained for a large number of the layers. Also, the optical confinement for the TE\textsubscript{01} mode in the core is increased (the loss is decreased) with the thickness of the gold layer and if the thickness of the cladding layers are computed with the quarter-wave stack condition in the case of infinite cladding pairs.

For the same wavelength, the real parts of the effective indices $\beta/k$ for the hollow-core Bragg fiber with or without gold layer are the same and for large number of layers can be approximated with the value given by the relation [16]:

$$\text{Re}(\beta/k) \approx \sqrt{n_1^2 - \left(\frac{J_{1,1}\lambda}{2\pi r_1}\right)^2}, \quad (45)$$

where $J_{1,1}$ is the same as in the relation (43). Thus, for $r_1 = 13.02$ μm, $n_1 = 1$ and $\lambda = 1.55$ μm, $\text{Re}(\beta/k) = 0.9973611820$, as in Tables 1–2. Also, for the same $r_1$ and $n_1$ but for $\lambda = 1.5292$ μm, $\text{Re}(\beta/k) = 0.9974316199$ as in Table 1. On the other hand, the imaginary part of the effective index $\beta/k$ is very sensitive to the number of the layers and if the structure is with or without a gold layer.

If the gold layer is located between the first and the penultimate layer, the loss for the same TE\textsubscript{01} mode is increased because the parts before and after the reflector gold layer of the fiber are decoupled.

Our method is in good agreement with the data known from the literature in the case of a hollow-core Bragg fiber without a gold layer. Thus for a hollow-core Bragg fiber with $N = 34$ layers (32 reflector layers, 16 pairs), $r_1 = 1.3278$ μm, $n_1 = 1, d_1 = 0.2133$ μm, $d_2 = 0.346$ μm, $n_2 = n_4 = ... = n_{34} = 1.49, n_3 = n_5 ... = n_{33} = 1.17, \lambda = 1$ μm, our effective index for the TE\textsubscript{01} mode $\beta/k = 0.8910672175 + 1.4226046712\times10^8 i$ is very close to the calculated value in Ref. [17], $\beta/k = 0.891067 + 1.4226 \times 10^8 i$.

When a high index material just before the outermost region of a hollow-core Bragg fiber is replaced by a gold layer, the optical confinement for the TE\textsubscript{01} mode in the core is increased about ten times. Thus, the light of a high power laser can be transmitted with very low loss due to the large confinement in the core of the fiber.
REFERENCES