THE STUDY OF THE TRANSIENT REGIME IN ELECTRIC CIRCUITS WITH EXCEL SPREADSHEETS

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Abstract. This paper focuses on the use of Excel spreadsheets in the study of the transient regime in electric circuits. Two relevant didactic tools, which have been created with the help of spreadsheets, are presented for the study of the transient regime in a circuit with resistance and capacitance connected in series. With the aid of the first tool we can simulate the charging and discharging of a capacitor. By changing the input data we can easily track how the charging and, respectively, the discharging curves change for the electric charge of the capacitor, the voltage across the capacitor and the current through the circuit. The significance of the time constant for the RC circuit is emphasized in graphs. The energetic characterization of the transient regime is done through a graph presenting the energy variation in time, from the capacitor, and the energy dissipated in the resistor. For a comparative analysis a graph is drawn, overlaying the curves of the time dependence in case of the capacitor voltage from three different RC circuits. The second tool has been conceived to process the data in the didactic experiment to study the process of discharging of a capacitor on a resistor. The voltage on the capacitor terminals measured during the discharging process is compared to the voltage calculated with the exponential variation law of the process. From the discharging curve we can determine the capacitance of the capacitor and, thus, certain experimental results are presented. The two previously described tools prove the facilities offered by spreadsheets in conceiving interactive Physics lessons. On the one hand, the possibility to simulate some physical phenomena is highlighted; on the other hand, it is shown that spreadsheets can be successfully used in the processing of experimental data.

Key words: spreadsheets, electric circuits, capacitor, physics education.

1. INTRODUCTION

Spreadsheets have recently become attractive educational tools for engineering and science through their wide spread and simplicity of spreadsheet programs [1]. The literature offers studies that describe how easy and functional Excel spreadsheets are. In this respect, examples have been given in the classroom of how to process data from tables [2], the enormous calculation capacity has been...
demonstrated through the functions at the users’ disposal [3, 4] and the rapid feedback to data change has been highlighted [5, 6].

In the process of improving the quality of the teaching-learning paradigm, spreadsheets, as both theoretical and practical tools, constitute a big challenge for teachers and students. There are authors who plead for a large-scale utilization of spreadsheets in the teaching of technical-scientific subjects [7, 8]. Next, we will give examples of papers that approach electric circuits and electromagnetism in general with the help of spreadsheets. These tools can be used to record and process data, to compare measurements, or for theoretical calculations and simulations.

It has been demonstrated that the spreadsheet represents an efficient modeling as it helps students in the process of learning about electric circuits [9]. Also, any teacher can apply different spreadsheet models for simulation. The user interface has been thoroughly presented and the didactic interest of such a simulation tool has been discussed, mentioning a pilot experiment in this field, as well [10]. It has been shown how the simulation with spreadsheets for a wide range of integrated digital circuits is characterized by low costs, flexibility, simplicity and the adequate character of this simulation for educational purposes has been discussed [11]. The learning opportunities on the digital signal processing have been explored using a spreadsheet and a specialized simulation software pack. Thus, application programs could be tested entirely by means of a software environment before being utilized on a target system in real time [12].

The manner of applying the technology of information (IT) has been illustrated with spreadsheets for the analysis, design and simulation of typical electric and electronic circuits. In this respect, the correlation between experiment and theory has been analyzed in order to support the laboratory lessons referring to electric circuits [13]. There has also been presented interaction of Excel with other programs such as Mathcad, PSpice, Matlab Simulink, OpenChoice in the teaching of some electronics classes [14]. The graphical simulation of analogical computers has been done with spreadsheets for the control systems. The method was based on the simulation of the basic blocks of alternative current which had been connected using an incorporated graphical interface [15].

Approaching the mathematical formalism of Physics with the help of spreadsheets offers a step-by-step means in the teaching and learning of numerical methods. This particular means increases students’ interest in learning [16]. Examples of applying numerical methods with spreadsheets in electromagnetism have been presented to determine the electric potential. The surfaces with potential have been graphically visualized [17] and there has been calculated the potential
for the electric charge distribution in a diode with junction as a demonstration of solving Poisson’s equation [18]. A software program for spreadsheets, using the power of 123 macros, has proven to be an important didactic tool in the numerical solving of electrostatic problems with boundary conditions. Following the feedback from change in the input data, students could have a better grasp of the mathematical model associated to the physical system [19].

Due to the fact that Physics laboratories from schools have certain limitations regarding equipments and measurement devices, didactic tools and virtual experiments have been suggested to lead to the understanding and assessment of Physics notions in a wide range of situations [20–24]. In this respect, besides the LabView graphics as a programming means, the MS Excel application has been used for the graphic representation of certain physical measures according to the frequency of the power voltage for the series RLC circuit of alternative current [25]. Also for the series RLC circuit a tool done with spreadsheets has been presented in order to calculate some specific measures and to graphically highlight the phase angle between the voltage and the current according to the input data [26].

This article presents two relevant didactic tools made with the help of interactive spreadsheets to study the transient regime in a circuit with resistance and capacitance connected in series. With the first tool, the capacitor charging and, respectively, the discharging process are simulated. These processes are graphically rendered through the time dependence of the voltage and the electric charge across the capacitor and, respectively, the current through the resistor. It is highlighted the time constant of the circuit with its physical significance. By fixing a time moment in the input data, we calculate, at that precise moment in time, the voltage, the electric charge and the current across the circuit, and the values of these measures are graphically accentuated. The processes of charging and discharging are energetically characterized by the graphs which render the variation in time of the energy from the capacitor and the energy dissipated in the resistor. The second tool is used to process the data in an experiment of capacitor’s discharge on a resistor. In the case of this didactic experiment described in literature, electrolytic capacitors of large capacitance are used as a voltage source the 9V battery, and the discharge is done across the resistance of a voltmeter [27]. The innovation consists on the way in which the data are processed with the Excel spreadsheets. The values measured for the voltage across the capacitor during the discharging process are compared to those calculated from the law of exponential variation of voltage in time. Thus, on the same graph, we have overlaid the theoretical curve and the curve obtained from the experimental data. From the
discharging curve we determine the electric capacitance of the capacitor, which is compared to the capacitance inscribed on the capacitor and the relative error is calculated.

2. EXCEL TOOLS

2.1. THE “RC SIMULATION” TOOL

The “RC Simulation” tool consists of several spreadsheets in interaction for the simulation of the charging, respectively, discharging process of a capacitor. We have utilized a construction procedure which is similar to the one used in other papers focusing on the exploitation of spreadsheets in the teaching and learning of Physics [26]. The main spreadsheet of the tool comprises the sections “Input data”, “Results” and the areas of the associated graphs grouped in two categories. The first category contains the time dependencies for the electric charge on the capacitor, the voltage across the capacitor and the current through the circuit. The second category is represented by the energetic graphs. The two energetic graphs describe the time variation of the energy from the capacitor and the energy dissipated by the resistor, in comparison with the work done by the source when charging, and with the initial energy from the capacitor when discharging. With the aid of the option “Freeze panel”, each of the graphs can be laid next to the input data in order to track the feedback to data change more easily. The graphs from the first category also emphasize the time constant of the circuit with its physical significance. Also, these graphs show the maximum values for the electric charge, the voltage across the capacitor and the current through the circuit.

![Fig. 1 – The main spreadsheet of the “RC_Simulation” Tool – Charging of capacitor. The colored versions can be accessed at http://www.infim.ro/rrp/](image-url)
Transient regime in electric circuits with Excel spreadsheets

Figure 1 renders the main spreadsheet in the case of the simulation of the capacitor charging process with the graph of the voltage across the capacitor according to time, placed next to the input data.

We describe the role of each section and the calculation formulae in Excel for certain physical measures that describe the charging, respectively, discharging process of the capacitor.

In the section “Input data” we introduce:

- Process type in cell B4 through the letter $i$ for charging and any other letter for discharging;
- Voltage of the source, $U_0$, expressed in volts (V), in cell B5. If we fix the discharging process in cell B4, then the value from cell B5 represents the initial voltage across the capacitor;
- Capacitance of capacitor, $C$, expressed in microfarads ($\mu$F), in cell B6;
- Resistance of the resistor, $R$, expressed in kilo-ohms (kΩ), in cell B7.

In the section “Results” there are displayed the values for:

- Time constant of the circuit, $\tau$, expressed in seconds, in cell B12;
- Electric charge across the capacitor, $q(t)$, in cell B14, voltage across the capacitor, $U_C(t)$, in cell B15, and the current through the circuit, $I(t)$, in cell B16, calculated at a certain moment in time, $t$, specified in the input data in cell B9. The electric charge is expressed in micro-coulombs ($\mu$C), the voltage across the capacitor in volts, and the current through the circuit in...
micro-amperes (μA). By establishing the type of process in cell B4, the three measures are calculated at the charging and discharging of the capacitor on the resistor.

- The energy accumulated in the capacitor at the moment \( t \), \( W_C(t) \), in cell B18, the energy dissipated across the resistor until the moment \( t \), \( W_R(t) \), in cell B19, the energy spent by the source until the moment \( t \), \( W(t) \), in cell B20, for the charging process. For the discharging process, in cell B18 we have displayed the energy remained in the capacitor at the moment \( t \), whereas in cell B20 we have displayed the sum between the energy from the capacitor and the energy which was dissipated across the resistor.

To do the calculations in Excel we name the cells in which we introduce data and some cells with results as follows: “Type_P” for cell B4, “Voltage_0” for cell B5, “Capacitance” for cell B6, “Resistance” for cell B7, “Constant_T” for cell B12.

First, we give as example the calculation of the electric charge at the time moment \( t \) fixed in the input data in cell B9. For this, we take into account that, when charging, the electric charge grows exponentially from 0, when \( t = 0 \), to the maximum value \( q_0 = CU_0 \), when \( t \to \infty \), according to an exponential law [28]:

\[
q(t) = q(0)(1 - e^{-t/RC})
\]  

When discharging, the electric charge decreases exponentially from \( q_0 \), when \( t = 0 \), to 0, when \( t \to \infty \):

\[
q(t) = q_0 e^{-t/RC}
\]  

Deduction of the relations (1) and (2) is largely treated in literature and needs no further emphasis [28–30].

With the previous cell names, based on the relations (1) and (2), the Excel formula to calculate the electric charge in cell B14 is:

\[
=\text{IF}(\text{Type_P}=\text{"I"};\text{Capacitance}*\text{Voltage_0}*(1-\text{EXP}(-\text{Time}/\text{Constant_T}));\text{Capacitance}*\text{Voltage_0}*\text{EXP}(-\text{Time}/\text{Constant_T}))
\]

We have used the logical function IF to do the calculation both in charging and in discharging, according to the process type established in the input data, in cell B4. The time constant of the circuit, \( \tau = RC \), is calculated in cell B12 with the formula “=Resistance*Capacitance*10^(-3)”, where we multiplied with \( 10^{-3} \) to express the result in seconds, considering that the resistance is expressed in kΩ and the capacitance in μF.

To calculate the voltage across the capacitor at the moment in time \( t \) in cell B15, we take into consideration the fact that, when charging, the voltage grows exponentially from 0, when \( t = 0 \), to \( U_0 \), when \( t \to \infty \):

\[
U_C(t) = U_0(1 - e^{-t/RC})
\]  

(3)
When discharging, the voltage across the capacitor decreases exponentially from $U_0$, when $t = 0$, to 0, when $t \to \infty$:

$$U_C(t) = U_0 e^{-t/RC}. \quad (4)$$

Transposing in Excel the relations (3) and (4) and using the logical function “IF”, we obtain a formula to calculate the voltage across the capacitor in cell B15, by analogy with the calculation formula for the electric charge from cell B14.

When charging and discharging, the current across the resistor, in the module, decreases exponentially from the maximum value $I_0 = U_0/R$, when $t = 0$, to 0, when $t \to \infty$. When discharging, the direction of the current through the resistor is contrary to the direction of the current through it during the charging of the capacitor [28]:

$$I(t) = I_0 e^{-t/RC}. \quad (5)$$

By transposing in Excel the relation (5) we get to the formula to calculate the current across the resistor in cell B16.

To calculate the energies $W_C(t)$, $W_R(t)$ and $W(t)$ we consider the energetic balance during the processes of charging and discharging of the capacitor on the resistor.

The energy from the capacitor at the moment in time $t$ is:

$$W_C(t) = \frac{1}{2} C U_C^2(t), \quad (6)$$

where the voltage $U_C(t)$ is given by the relation (3) at charging and by relation (4) at discharging.

The energy dissipated through the resistance $R$ until the moment $t$ is:

$$W_R(t) = \int_0^t R \cdot I_C^2(t) \, dt' = \frac{1}{2} C U_0^2 \left(1 - e^{-2t/RC}\right). \quad (7)$$

By transposing in Excel the relations (6) and (7) we obtain the formulae to calculate the energies $W_C(t)$, $W_R(t)$ in cells B18 and B19. In cell B20, to calculate the energy $W(t)$, the sum of the results from cells B18 and B19 is done.

Figure 2 renders the main spreadsheet of the tool in which the input data specified in Fig. 1 are kept, but the discharging process was established in cell B4. The graphs for the time dependence of the current across the circuit resemble those from Fig. 3 and have not been presented.

The source tables of the graphs are placed in the secondary sheets of the tool.

The source table for the first set of graphs contains the column of the moments in time and the columns in which there are calculated: the electric charge across the capacitor, $q(t)$, the voltage across the capacitor, $U_C(t)$, and the current through the circuit, $I(t)$. Moreover, supplementary columns have been used for the series of data which emphasize, on the graphs, the time constant of the circuit and the line with which we physically interpret the time constant. Other additional
columns have been used for the maximum values of the charge, voltage, current and marking the time moment introduced in the input data, associated with the values calculated for the three measures dependent on time.

In order to generate the time moments we have utilized an increasing series, \( n \), of a unit step in column A of the spreadsheet and a time interval \( \Delta t = 6 \cdot \tau \), where \( \tau = RC \) is the time constant of the circuit. We have considered a time quantum equal to the 200\(^{th} \) part of the \( \Delta t \) interval. Thus, with the series \( n \) starting from 0 in cell A4, the starting value for the time moments in column B is calculated with the Excel formula \( ^{\text{""}}=A4*(((6*Constant_T)/200) \rangle \). The formula is propagated in column B up to cell B204 corresponding to \( n=200 \) from column A.

In order to calculate the values of the electric charge in column C we consider the relations (1) and (2). With the help of the logical function IF we write in cell C4 the Excel formula:

\[ ^{\text{""}}=\text{IF}(\text{Type}_P="I";\text{Voltage}_0*\text{Capacitance}*(1-\text{EXP}(-B4/Constant_T));\text{Voltage}_0*\text{Capacitance}*\text{EXP}(-B4/Constant_T)) \]

This formula is propagated along column C up to cell C204.

In an analogous manner, we act to calculate the values of the voltage across the capacitor and the current through the circuit. The structure of the source tables resembles the one used by the authors in other papers [26].

![Fig. 3 – The energetic graph for the charging of capacitor. The colored versions can be accessed at http://www.infim.ro/rrp/](image-url)
In each graph, the time dependences $q(t)$, $U_C(t)$ and $I(t)$ are marked by a continuous curve colored in red; the time constant together with the associated values are marked by a segmented line colored in green, whereas the maximum values for the charge, voltage and current with a dotted line colored in brown. The significance of the time constant is highlighted by the line marked continuously and colored in blue. Thus, in the graph from Fig. 1, when charging, it can be observed how the time constant represents the time according to which the charge across the capacitor would reach the maximum value, $q_0$, if it grew with the same variation speed as in the initial moment. In the graph from Fig. 2, when discharging, it can be observed how the time constant represents the time according to which the charge across capacitor becomes equal to zero if it decreases with the same variation speed as in the initial moment.

From all the energetic graphs, we render the one describing the charging of the capacitor. This particular one is presented in Fig. 3 with the input data from Fig. 1. The time constant, $\tau$, is also marked with a dotted green line on the graph, with the associated values $W_C(\tau)$, $W_R(\tau)$ and $W(\tau)$. The source tables for the energetic graphs are built analogously to the ones of the graphs from the main spreadsheet considering, though, the relations (6) and (7).

The energy consumed by the source for charging until the moment $t$ is:

$$W(t) = W_{C(t)} + W_{R(t)} = C U_0^2 \left(1 - e^{-t/\tau} \right). \quad (8)$$

For the entire charging process, putting $t \to \infty$, and considering relations the (6), (7) and (8) we obtain:

$$W_C = W_R = (1/2) C U_0^2 \quad W = C U_0^2. \quad (9)$$

In relations (9), $W_C$, $W_R$ and $W$ represent the energy accumulated in the capacitor, the energy dissipated in the resistor and, respectively, the energy consumed by the source in the charging process.

The curves $W_C(t)$ and $W_R(t)$, traced according to the relations (6) and (7) are marked on the energetic graph with continuous blue and pink lines, while the curve $W(t)$ traced according to relation (8) is marked in a continuous red line. The values $W_C$, $W_R$ and $W$ are marked on the graph with a dotted blue and respectively red line.

It can be observed that, at the beginning of the charging process, the curve $W(t)$ overlays the curve $W_R(t)$, which means that most of the work done by the source dissipates across the resistor, and only a small part contributes to the charging of the capacitor. Still, as the charging process advances, the capacitor assumes more and more energy from the voltage source, and less and less energy dissipates across the resistor. At the end of the process, when $t \to \infty$, the curves $W_R(t)$ and $W_C(t)$ overlay, tending asymptotically towards a value of the energy equal to half of the energy consumed by the source, according to relations (9). Therefore, during the process of charging the capacitor through the resistor, under
constant voltage, independently of the values of the resistance and the capacitance, half of the work done by the source is lost in the resistor, while the other half is stored in the capacitor [28].

A comparative analysis of the charging and discharging processes is rendered in a special spreadsheet. The comparison is made by overlapping the curve representing the time dependence of the voltage across the capacitor from three RC circuits. This spreadsheet is rendered in Fig. 4, for the discharging process. The type of process is fixed in the main spreadsheet, and the other input data, such as the voltage of the sources, the capacitances of the capacitor and the resistances of the resistors are introduced in the table situated on the left of the graph. In the table from Fig. 4, for the three RC circuits, we have considered the same value of the resistance, \( R \), but we have different values for the voltage \( U_0 \) and capacitance \( C \). By modifying the data from the table, we can track the graphic response on the comparative graph. Under this table there are displayed the calculated values of the time constants for the three RC circuits. The source table of the graph is placed in a secondary sheet. To generate the time moments we have applied a procedure similar to that from the source tables of the graphs from the main spreadsheet. The difference consists in choosing the time quantum as the 200\(^{th}\) part of the interval \( \Delta t=6\cdot\tau' \), where \( \tau' \) represents the maximum value of the time constants of the three RC circuits.

2.2. THE “RC EXPERIMENT” TOOL

The “RC experiment” tool has been done in order to process the data from an experiment consisting of the discharge of an electrolytic capacitor. After the
capacitor is charged at a source of constant voltage, it is detached from the source and it is connected to a voltmeter. The discharging takes place on the resistance of the voltmeter. At the same time, with the aid of the voltmeter we measure the voltage across the capacitor. The connection of the capacitor to the source during charging, and then to a voltmeter during discharging, is done by following the polarity each time. Measurements are taken until the capacitor discharges completely.

Figure 5 renders the spreadsheet in which experimental data are introduced. An example is given: a capacitor of 100 \(\mu F\) charged with a battery of 9 V and then discharged on a voltmeter with the resistance of 1200 k\(\Omega\). The input data are: the capacitance inscribed on the capacitor, \(C\), in cell C5, the resistance of the voltmeter, \(R\), in cell C6 and the voltages measured with the voltmeter at equal time intervals, \(U_C(t)\). The values of the voltage have been recorded at every 20 seconds and they have introduced in column C of the second data table beginning with cell C9. The capacitance is expressed in microfarads (\(\mu F\)), the resistance in kilo-ohms (k\(\Omega\)) and the voltage in volts (V). The figure presents the first 20 values of the voltage measured. Cells C5 and C6 are named “Capacitance”, respectively “Resistance”, while cell C9 is named “Voltage_0”, in it which we introduce the voltage across the capacitor at the moment \(t = 0\), \(U_0\), when the capacitor is detached from the voltmeter.

![Spreadsheet for the tool “RC Experiment” in which data from measurements are introduced.](http://www.infim.ro/rrp/)
The experimental determination of the time constant, $\tau$, allows the calculation of the electric capacitance $C$ of the capacitor [27]. By logarithm and relation (4) we obtain:

$$\ln\left[\frac{U_0}{U_C(t)}\right] = \frac{1}{RC} \cdot t. \tag{10}$$

Based on relation (10), where $U_C(t)$ represents the value of the voltage across the capacitor measured at a certain moment in time $t$, through the graphic representation $\ln[U_0/U_C(t)]$ according to $t$, we obtain a line with the slope:

$$\tan \alpha = \frac{1}{RC} \tag{11}$$

From the slope of the line we can determine the electric capacitance of the capacitor.

The processing of the experimental data is done in a table from the secondary sheet presented in Fig. 6. In this table, from the data input sheet, the time moments are imported in column B, whereas the values of the measured voltage, $U_C(t)$, are imported in column C. In column D the values $\ln[U_0/U_C(t)]$ are calculated, while in column E the linear regression to data from column D is applied to explore relation (10). In column F the values of the voltage across the capacitor are calculated at the time moments from column B with relation (4), using the input data for the capacitance, resistance and initial voltage. The field with the values of the moments in time from column B is named “Time”, the field with the values of the measured...
voltage from column C is named “Voltage”, and the field with the values \( \ln\left[\frac{U_0}{U_C(t)}\right] \) from column D is named “Logarithm_U”.

To apply the linear regression to data from column D, we write the following Excel formula in cell E4:

\[
=\text{B4} \times \text{SLOPE(Logarithm\_U;\text{Time})} + \text{INTERCEPT(Logarithm\_U;\text{Time})}
\]

The formula is propagated along column E up to the cell on the same line from column C in which there is displayed the last value of the measured voltage.

The function SLOPE calculates the slope of the straight line obtained from the linear regression, while the function INTERCEPT calculates the intercept.

Figure 7 renders the spreadsheet with the results of the experimental processing. The results are compared to the input data, the associated graphs being placed under the two tables.

In the first graph there is traced the curve of the voltage variation in time according to equation (4), marked with a continuous red line, as well as the curve given by the experimental values marked with blue dots. The time constant of the circuit, with an associated voltage value, is also highlighted on the graph with a dotted green line. The second graph renders the curve \( \ln\left[\frac{U_0}{U_C}\right] = f(t) \), and from its slope we determine the capacitance of the capacitor. The blue dots represent the data obtained from measurements, while the red line is obtained from the linear regression of the data.
To calculate the capacitance $C_m$ from the experimental data we take into account relations (10)-(11). With the help of the function SLOPE we write the following Excel formula in cell N3:

```
="1000/(Resistance*SLOPE(Logarithm_U;Time))"
```

To express the capacitance $C_m$ in $\mu$F we have multiplied by 1000 as the resistance $R$ is expressed in k$\Omega$.

In cell N4 the relative error is calculated for the measured capacitance expressed as a percentage.

3. CONCLUSIONS

Spreadsheets can be successfully integrated in Physics lessons to simulate certain physical phenomena and to process experimental data.

By changing the input data, with the first tool we are able to analyze different particular cases for the simulation of the charging and discharging of a capacitor through a resistor. The graphic representations of the physical measures dependent on time facilitate the understanding of the phenomena that determine the transient regime in electric circuits. The tool can be easily adapted for the study of the transient regime in a circuit with coil and resistance (RL). The second tool offers the comparative analysis between theory and experiment. The experimental data are rapidly processed and thus is checked the law of variation in time of the voltage across the capacitor in the discharging process.

With the help of the two tools the quality and experience of learning can be improved. Classroom use can lead to increased academic performance and motivation of students to study Physics.

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REFERENCES