USING MICROSOFT EXCEL IN TEACHING AND LEARNING RELATIVISTIC KINEMATICS

I. GRIGORE\textsuperscript{1,2}, CRISTINA MIRON\textsuperscript{1*}, E. S. BARNA\textsuperscript{1}
\textsuperscript{1}Faculty of Physics, University of Bucharest, Romania
\textsuperscript{2}“Lazăr Edeleanu” Technical College, Ploiești, Romania
E-mail: cmiron_2001@yahoo.com
Received August 1, 2013

Abstract. In this paper there are presented three instruments developed with Microsoft Excel that may be helpful in the process of teaching and learning the relativistic kinematics. The instruments devised complete other educational software that offers support to students in their efforts to overcome obstacles encountered when learning relativistic phenomena. The first instrument, named “Analysis of events”, is used to the study of the space-time relationship between two events located in two inertial reference frames that are in relative movement one to the other. Particularly, this instrument helps us to check time dilation and simultaneity of the events, as well as the invariance of the space-time interval between them. The second instrument is used for the relativist composition of the velocities, whereas the third calculates length contraction of a linear body in an inertial reference frame that is in motion to its own reference frame. The construction of these instruments is thoroughly explained together with the functions at the disposal of the Excel users, and the way in which these can be used in various situations. There are also discussed the advantages of using spreadsheets in the modeling of physical phenomena, especially of the relativistic ones.

Key words: spreadsheets, Microsoft Excel, relativistic kinematics, Lorentz transformations, physics.

1. INTRODUCTION

Technological innovations significantly alter the way in which scientific research is carried out, and their usage becomes efficient only if there is a corresponding change in the teaching methods for science in school [1]. Consequently, there has appeared a series of works that analyze the effective use of technology in the scientific and mathematical field, also presenting examples of technological products available for students’ training in a new context of a society in the midst of technical and scientific development.
A certain category of products addresses students’ training in the theory of relativity considering the challenges met by them when studying this chapter of Physics.

A series of researches carried out by questioning batches of future Physics teachers has shown that a major obstacle in the study of the theory of relativity is represented by several concepts that make up the frame of relativistic kinematics, such as the following: reference frame, event and simultaneity in time. These studies that are part of a research domain still fairly unexplored [2] have shown that students mistake the event for its perception by the observer and they believe that simultaneity is absolute and independent of the relative movements [3]. Also, other profound misunderstandings regarding the space-time structure of relativistic kinematics have come to surface. One should note the study carried out by Gamze Sezgin Selcuk which showed that students have difficulties in understanding the concepts of time dilation and length contraction [4], which constitutes in its turn obstacles in understanding other relativistic concepts.

Due to the fact that relativist Physics is not part of our direct experience, the traditional learning methods are more often than not abstract and based on Mathematics [2]. As a consequence, the shift from classic kinematics to the relativistic one demands a radical change of the conceptual frame. In this respect, several soft products specialized in lessons on the theory of relativity have been placed at students’ disposal. One of the directions to follow was the elaboration of programs of computer graphics to simulate some relativistic effects, some of the programs consisting in 3D computer games since more and more people are feeling comfortable in the virtual worlds generated by computer games [5–7]. Similar to these is the software that consists of interactive lessons on the theory of relativity, and these are grouped in sections from rendering certain relativistic phenomena on assessment tests [8].

Another direction to follow is the exploration of Excel spreadsheets and considering the popularity of this program we have approached this point of view in this paper.

Spreadsheets have been already used to treat certain physical phenomena or models in a series of articles published in specialty magazines, such as the study of planetary orbits, the simulation of electric circuits and of Young’s experiment, the investigation of the Compton effect, the study of the wave functions for the quantum harmonic oscillator etc [9–14]. In the field of relativistic Physics, in the work of S. R. Carson, spreadsheets are used to transform the electromagnetic field when changing the reference frame [15], whereas in the work of C. Iyer in collaboration with G. M. Probhu, they are associated with the numeric calculus in a general approach to Lorentz transformations in the two-dimensional space in which there is also included the spatial rotation alongside the conventional Lorentz
transformation [16]. In all these works, there are explored both the mathematical capacities of the spreadsheet to model physical phenomena, and the graphic facilities through diagrams associated to the results obtained.

Excel can be preferred as a program due to its flexibility and large-scale usage in the present society, as any student, almost certainly, will have access to a spreadsheet in his or her future career [17].

Among the advantages offered by spreadsheets, there are the user-friendly interface with the cell structure easy to manipulate and the large number of functions at the user’s disposal, many of which can be used in applications from Physics [14, 18]. Other advantages are the immediate feedback to the change of data or used formula [19] as well as the processing of large quantities of data without having to program [14].

The present paper presents three instruments made up by Excel spreadsheets that can be used in the process of teaching and learning some consequences of the Lorentz transformations in the relativistic kinematics. This paper continues the study of the authors exploring spreadsheets in the teaching of Lorentz transformations within the special theory of relativity [19].

With the help of the first instrument, named “Analysis of events”, we can check time dilation, relativity of simultaneity and the invariance of the space-time interval defined with the metric of the Minkovski space. Also, with this instrument we can follow the time order and the space positioning of two events in two different inertial reference frames in relative movement one to another, and we can establish the interval type between events according to the space-time coordinates of the events expressed in one of the reference frames.

With the help of the second instrument we can do the relativistic composition of velocities. Knowing the velocity of a material point in an inertial reference frame we can find out the velocity of the same material point in another inertial reference frame which is in relative movement to the first. This instrument also offers a comparative diagram for the components of velocities of the particle in the two reference frame according to the three axes of coordinates and the components of velocities of a reference frame in relation to another according to the same axes of coordinates.

We use the third instrument for the calculus of length contraction of a linear body in relation to an inertial reference frame that moves from its own reference frame. Also, this instrument renders the graphic of the kinematical length of the body according to the velocity of the mobile reference frame in relation to its own reference frame.

With the help of each of the three instruments we can check that at small velocities compared to the one of light in vacuum, the relations from relativistic kinematics are reduced to the relations from classic kinematics.
2. THEORETICAL BACKGROUND

2.1. POSTULATES OF SPECIAL RELATIVITY – LORENTZ TRANSFORMATIONS

The Special Relativity Theory (SRT) is based on the two postulates enunciated by Albert Einstein, the first one generalizing the principle of relativity from classic mechanics, whereas the second represents the experimental conclusion that the speed of light in vacuum is the maximum propagation speed of any physical interaction:

I. Physical phenomena happen identically in all inertial reference frames, if the initial conditions are identical.

This postulate, also entitled the relativity postulate, leads to the equivalence of inertial reference frames in relation to all physical phenomena, therefore we can say that the laws of physics are the same for any inertial reference frame. Thus, the first postulate represents the generalization of Galileo’s relativity principle from mechanics to the entire physics.

II. The speed of light in vacuum has the same value in all directions in all inertial reference frames.

In other words, the speed of light does not depend on the movement of the light source or the observer. When it was first issued, the constancy postulate for the speed of light could not be checked, the decisive experimental confirmation occurred only in the second half of the 20th century. Since this postulate makes us radically reconsider the ideas of space and time, the resistance met throughout many years after the apparition of the relativity theory is perfectly explainable [20].

Considering the two postulates, space-time coordinates of an event $E$ change when we move from an inertial reference frame $S$ to another inertial reference frame $S'$ after the Lorentz transformations. We consider by convention frame $S$ as the fixed one, and $S'$ as the mobile frame in relation to $S$.

If the mobile frame $S'$ moves in any direction so that the axes of the two frames are parallel, then we obtain Lorentz transformations in a general form [20]:

\[ \vec{r}' = \vec{r} + \frac{c^2}{v^2} \left( \vec{r} \cdot \vec{v} \right) \vec{v} - \gamma \vec{v} t, \]

\[ t' = \gamma \left( t - \frac{\vec{r} \cdot \vec{v}}{c^2} \right), \]

where $\vec{r}$ is the position vector of the event in the reference frame $S$, $\vec{r}'$ the position vector of the event in $S'$ and $\vec{v}$ is the speed vector of the frame $S'$ to $S$, $t$ is the moment in time of the event $E$ in $S$, and $t'$ the moment in time of the same event in $S'$.

In equations (1)–(2) the term $\gamma$ (the Lorentz factor) are given by the relation:
\[ \beta = \frac{v}{c}; \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}, \quad (3) \]

where \( c \) is the speed of light in vacuum.

If the speed of the reference frame \( S' \) to the reference frame \( S \) is very small in comparison to the speed of light in vacuum, then Lorentz transformations are reduced to Galileo transformations from classic mechanics.

### 2.2. THE CONSEQUENCES OF THE LORENTZ TRANSFORMATIONS

Lorentz transformations have a series of consequences among which we can name length contraction, time dilation, relativity of simultaneity, the relativistic composition of velocities etc. we will further present these consequences, emphasizing the main ideas:

**Length contraction.** We consider a linear shaped body (a ruler, for example) on the \( Ox \) axis, in a state of rest in the inertial reference frame \( S \), named its own reference frame. The length of the body in its own reference frame is marked with \( L_0 \) and it is called proper length. In the mobile frame \( S' \) which moves with a constant velocity \( v \) from \( S \) so that the vector \( v \) is parallel with the \( Ox \) axis, the length of the same body, called the kinematical length, marked with \( L' \) is smaller than the static length, given the equation:

\[ L' = L_0 \sqrt{1-\beta^2}, \quad (4) \]

where we marked with \( \beta \) the ratio between the velocity \( v \) of \( S' \) to \( S \) and the speed of light in vacuum, \( \beta = v/c \).

**Time dilation.** We consider a process localized in a point in the inertial reference frame \( S \) in which the observer is in the state of rest. The time interval of the process in relation to the frame \( S \) (in relation to its own reference frame) is called the static time interval and it is marked with \( \Delta t_0 \). In the reference frame \( S' \), which moves from \( S \) with the constant velocity \( v \) along the axis \( Ox \), the same process occurs in a bigger time interval \( \Delta t' \), called kinematical time interval, given the equation:

\[ \Delta t' = \Delta t_0 \frac{1}{\sqrt{1-\beta^2}}. \quad (5) \]

**Relativity of simultaneity.** We consider two simultaneous events in the frame \( S \) (occurring at the same moment in time \( t_1 = t_2 \)) but in different places \( (x_1 \neq x_2) \). In the mobile frame \( S' \) that moves with a constant velocity \( v \) along the \( Ox \) axis from \( S \), the two events are no longer simultaneous, but separate by a time interval given by the equation:
\[ \Delta t' = -\frac{v}{c^2} \frac{\Delta x}{\sqrt{1 - \beta^2}}. \]  

The relativistic composition of velocities. If the frame S' moves from S in any direction with the constant velocity \( v_0 \) then the velocity vector \( \vec{v}' \) is function of the velocity vector \( \vec{v}_0 \) according to the equation [20]:

\[ \vec{v}' = \frac{-\vec{v} + \frac{1}{v_0^2}(\gamma - 1)(\vec{v} \cdot \vec{v}_0)\vec{v}_0 - \gamma \vec{v}_0}{\gamma \left(1 - \frac{v_0^2}{c^2}\right)}. \]

In the relation (7) we wrote:

\[ \gamma = \frac{1}{\sqrt{1 - \beta_0^2}}; \quad \beta_0 = \frac{v_0}{c}. \]

It is observed that if the velocity \( v_0 \) is very small compared to the speed of light in vacuum, \( c \), then we obtain the rule of the classic composition of velocities:

\[ \vec{v}' = \vec{v} - v_0. \]

Other notable consequences of the Lorentz transformations such as the relativistic Doppler Effect and the Thomas precession are presented in the specialty literature [20, 21].

2.3. THE MINKOWSKI SPACE

Considering the three-dimensional physical space, any event that occurred in any of its points and at a certain moment in time, can be considered as a point of a fictitious space, quadric-dimensional, marked with \( E_4 \), called the Minkowski space.

Consequently, the Minkowski space is the space of events and it has three spatial dimensions and a temporal one, namely \( x_1 = x, x_2 = y, x_3 = z, x_4 = it \), where for the homogenization of dimensions time is multiplied with the speed of light, and \( i = \sqrt{-1} \) represents the imaginary unit.

In this case, the Lorentz transformations represent a transformation of the coordinates of the events in the space \( E_4 \).

We define the space-time interval, \( s^2 \), that separates two events \( E_1(x_1, y_1, z_1, t_1) \) and \( E_2(x_2, y_2, z_2, t_2) \) related to the reference frame S as being the physical quantity given by the relation:

\[ s^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - c^2 (\Delta t)^2, \]

where \( \Delta x = x_2 - x_1, \quad \Delta y = y_2 - y_1, \quad \Delta z = z_2 - z_1, \quad \Delta t = t_2 - t_1. \)
For some events related to the reference frame \( S' \), the space-time interval, \( s'^2 \), is

\[
s'^2 = (\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2 - c^2 (\Delta t')^2,
\]

where \( \Delta x' = x_2' - x_1' \), \( \Delta y' = y_2' - y_1' \), \( \Delta z' = z_2' - z_1' \), \( \Delta t' = t_2' - t_1' \).

The space-time interval between two events is an invariant Lorentz quantity, namely it has the same value in all inertial reference frames:

\[
s^2 = s'^2 = \text{inv}.
\]

The invariance of the space-time interval between two events is easily demonstrated by writing its expression in the inertial reference frame \( S \) considered fixed and in the mobile frame \( S' \) which moves from \( S \) with the constant velocity \( v \) and considering the Lorentz transformations [20–22].

We can classify the space-time intervals in space-like interval for \( s^2 > 0 \), time-like interval for \( s^2 < 0 \) and light-like interval for \( s^2 = 0 \). The properties of three types of space-time intervals are given in the specialty literature [20–22].

The very useful notion of luminous hypercone for the classification of the events with the geometrical interpretation from the Minkovski space in which it is divided in the regions of absolute past, absolute future and the region of events absolutely distant in relation to the original event from the top of the cone is debated in the specialty literature and it is not presented [20–22].

### 3. EXCEL INSTRUMENTS

#### 3.1. THE “ANALYSIS OF EVENTS” INSTRUMENT

The instrument “Analysis of events” consist of a spreadsheet in which we can analyze the spatial-temporal events between two events \( E_1 \) and \( E_2 \) in which two inertial reference frames in relative motion one to another. By convention we will mark with \( S \) the fixed reference system and with \( S' \) the mobile reference frame. In particular, we can use this instrument to analyze the relativity of simultaneity and time dilation.

The procedure of file construction resembles the one used in the authors’ paper referring to the utilization of spreadsheets in the teaching of Lorentz transformations [19]. Thus, the spreadsheet is made from two sections, namely “Data input” and “Results”. We describe the role of each section and the calculus formula in Excel in order to establish the space-time relations between events.

In the section “Data input” the following data are introduced: speed of light in vacuum, \( c \), expressed in km/s, the space-time coordinates for two events \( E_1 \) and \( E_2 \) in the fixed reference frame \( S \) and the velocity of the frame \( S' \) in relation to \( S \) under the form of ratios between the components of the velocity vector \( \vec{v} \) along the axes \( O_x, O_y, O_z \), and the speed of light in vacuum, \( \beta_i = v_i/c \), \( i = x, y, z \).

In Fig. 1 the image of the spreadsheet with the two sections is rendered.
The section “Results” consists of two subsections, namely:

In the first subsection the space-time coordinates are calculated according to the data input of the two events $E_1$ and $E_2$ in the mobile reference frame $S'$. For this, we transpose in Excel the general Lorentz transformations for each set of space-time coordinates of the two events according to the ratios $\beta_i = \frac{v_i}{c}, i = x, y, z$ [19]. We have:

$$x_i^{(k)} = x_i^{(k)} + \frac{\beta_i}{\beta^2} \left( \gamma - 1 \right) \sum_{j=1}^{3} x_j^{(k)} \beta_j - \gamma \beta_i c t_j^{(k)}, \quad (13)$$

$$t^{(k)} = \gamma \left[ t^{(k)} - \frac{1}{c} \sum_{j=1}^{3} x_j^{(k)} \beta_j \right], \quad (14)$$

where the index $i$ corresponds to the components according to the three axes $I = x, y, z$ and the index $k$ corresponds to the event $E_1$, respectively $E_2$, therefore $k = 1, 2$.

![Fig. 1 – The spreadsheet for the instrument “Analysis of events”. The colored versions could be accessed at http://www.rrp.infim.ro/rrp/](image-url)
In order to transcribe in Excel the formula (13)–(14) for the calculus of the coordinates of the two events $E_1$ and $E_2$ in the frame $S'$ we name the cells in which data are introduced: “Speed_L” for cell D5, “Coord_x1” for cell B9, “Coord_y1” for cell C9, “Coord_z1” for cell D9, “Coord_t1” for cell E9, “Coord_x2” for cell B10, “Coord_y2” for cell C10, “Coord_z2” for cell D10, “Coord_t2” for cell E10, “Beta_x” for cell B14, “Beta_y” for cell C14 and “Beta_z” for cell D14.

With the previous names of the cells and marking the cells in which we calculate the quantities $\beta$ and $\gamma$ with Beta, respectively Gamma, the relations (13)–(14) for the calculus of the coordinates of the event $E_1$ in $S'$ are written in Excel as follows:

- **For the coordinate space $x'$**:
  \[
  \text{IF}(\text{Beta}<1;\text{Coord}_x1+(\text{Beta}_x/\text{Beta}^2)*(\text{Gamma}-1)*(\text{Coord}_x1*\text{Beta}_x+\text{Coord}_y1*\text{Beta}_y+\text{Coord}_z1*\text{Beta}_z)-\text{Gamma}*(\text{Coord}_x1*\text{Beta}_x+\text{Coord}_y1*\text{Beta}_y+\text{Coord}_z1*\text{Beta}_z);"\text{NO}")
  \]

  Similarly are written in Excel the formulas for the spatial coordinates $y'$ and $z'$.

- **For the coordinate time $t'$**:
  \[
  \text{IF}(\text{Beta}<1;\text{Gamma}*(\text{Coord}_t1-(1/(\text{Speed}_L*1000))*\text{Coord}_x1*\text{Beta}_x+\text{Coord}_y1*\text{Beta}_y+\text{Coord}_z1*\text{Beta}_z);"\text{NO}")
  \]

We utilize the logical function IF for the calculus of the space-time coordinates in $S'$ to return the message “NO” in the situation in which the user introduces from the keyboard values for the components of the velocity vector $\vec{v}$ expressed by the ratios $\beta_i$ ($i = x, y, z$) that lead to a module of the velocity of $S'$ to $S$ bigger than the speed of light in vacuum. In other words, the error of data input is signaled.

Analogously, there are written in Excel the relations for the calculus of the coordinates of the event $E_2$, with the difference that we replace in formula the index 1 from the notations of the coordinates with index 2.

In the second subsection the two events are compared in the fixed frame $S$ and in the mobile frame $S'$ by reporting in time and space. For this we calculate the time interval between the two events and the spatial distance between them in the two reference frames $S$ and $S'$:

\[
\Delta t = t_2 - t_1; \quad \Delta t' = t'_2 - t'_1, \quad (15)
\]
\[
\Delta r^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2, \quad (16)
\]
\[
\Delta r'^2 = (x'_2 - x'_1)^2 + (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2. \quad (17)
\]

From the point of view of the temporal comparison we can have the following three situations in $S$:

- “$E_1$ before $E_2$” for $\Delta t > 0$;
- “$E_1$ simultaneous with $E_2$” for $\Delta t = 0$;
- “$E_1$ after $E_2$” for $\Delta t < 0$. 
From the point of view of the spatial comparison we can have the following two situations in S:

- \( E_1 \) in the same place with \( E_2 \) for \( \Delta r = 0 \);
- \( E_1 \) in a different place from \( E_2 \) for \( \Delta r \) different than zero.

Analogously, we have the situations described in S’ also, using for comparisons the intervals \( \Delta r' \), respectively \( \Delta r' \).

With the help of the logical function “IF” from Excel the results are returned for the comparison according to the coordinates of the events in the respective cells for this section. Thus, with the notations utilized in the spreadsheet, the formula used to return the results of the space-time comparison in the frame S, are:

**The space comparison**

IF(AND(Coord_x1=Coord_x2,Coord_y1=Coord_y2,Coord_z1=Coord_z2),"E1 in the same place with E2","E1 in a different place from E2")

**The time comparison**

IF(Coord_t1<Coord_t2,"E1 before E2", IF(Coord_t1=Coord_t2,"E1 simultaneous with E2","E1 after E2"))

We utilized the logical function IF twice and the function AND considering the three situations to return the result.

Similarly are written in Excel the formulas for return the results of the space-time comparison in the frame S’.

Also in this section it is calculated the space-time interval that separates events, \( s \), with the formula (10) in S and with the formula analogous to the new coordinates in S’, and at the end of the section the result is returned for the space-time interval, meaning the space-like, time-like or light-like interval, according to the coordinates of the two events.

To transpose in Excel the calculus relations for \( s^2, s'^2 \), respectively for the type of interval, we name the cells in which we calculate \( \Delta r, \Delta t, s^2, \Delta r', \Delta t' \):

- \( \text{Distance}_S \) for cell D28, \( \text{Time}_S \) for cell D29, \( \text{Interval}_ST \) for cell F30, \( \text{Distance}_SP \) for cell D33 and \( \text{Time}_SP \) for cell D34.

We obtain the following formulas:

- For the calculus of the spatial interval and the time interval between events in the frame S:
  \[ \Delta r: \sqrt{(\text{Coord}_x2-\text{Coord}_x1)^2+(\text{Coord}_y2-\text{Coord}_y1)^2+(\text{Coord}_z2-\text{Coord}_z1)^2}; \]
  \[ \Delta t: \text{Coord}_t2-\text{Coord}_t1. \]

- For the calculus of the space-time intervals between events in the two reference frames:
  \[ s^2: \text{Distance}_S^2-(\text{Speed}_L*1000)^2*\text{Time}_S^2 \]
  \[ s'^2: \text{Distance}_SP^2-(\text{Speed}_L*1000)^2*\text{Time}_SP^2. \]

- For to return the type of space-time interval:
  IF(Interval_ST=0;"LIGHT-LIKE";IF(Interval_ST>0;"SPACE-LIKE";"TIME-LIKE").)
For this formula we used the logical function IF twice, considering we have three answer possibilities corresponding to the three types of space-time interval, light-like interval, space-like interval, and time-like interval.

Modifying the coordinates of the events in S, it will be checked the invariance of the space-time interval observing the same values both in the reference frame S, and in S′, in limit of the calculation errors in Excel.

**Example 1 (Relativity of simultaneity).** We consider two simultaneous events in the inertial reference frame S and we will show that the two events are no longer simultaneous in the mobile reference frame S′.

For example, the generation of two particles having in the frame S the spatial-temporal coordinates: \(E_1: x_1 = 1 \text{ m}, y_1 = 2 \text{ m}, z_1 = 0.5 \text{ m}, t_1 = 2 \text{ s}; E_2: x_2 = 2 \text{ m}, y_2 = 2 \text{ m}, z_2 = 0.5 \text{ m}, t_2 = 2 \text{ s}.

As observed, the second particle is simultaneously generated but at a distance of 1 m on the Ox axis from the first particle in the frame S, having in exchange the same coordinates on the Oy and Oz axes.

The mobile frame S′ moves from S with the constant speed in any given direction so that the vector \(\vec{v}\) has the components \((v_x, v_y, v_z) = (0.90 \, c; 0.10 \, c; 0.20 \, c)\).

The comparison between events obtained with the help of the spreadsheet in the two reference frames is presented Tables 1–3:

**Table 1**
The values of the space-time comparative analysis of the events \(E_1\) and \(E_2\) in the reference frame S for example 1

<table>
<thead>
<tr>
<th>Comparison between events in S</th>
<th>(\Delta r [\text{m}])</th>
<th>(\Delta t [\text{s}])</th>
<th>(s^2 = \Delta r^2 - c^2 \Delta t^2 [\text{m}^2])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reporting in space</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reporting in time</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Space-time interval</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


**Table 2**
The values of the space-time comparative analysis of the events \(E_1\) and \(E_2\) in the reference frame S′ for example 1

<table>
<thead>
<tr>
<th>Comparison between events in S′</th>
<th>(\Delta r′ [\text{m}])</th>
<th>(\Delta t′ [\text{s}])</th>
<th>(s^2 = \Delta r'^2 - c^2 \Delta t'^2 [\text{m}^2])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reporting in space</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reporting in time</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Space-time interval</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


**Table 3**
The return of the type of space-time interval between events \(E_1\) and \(E_2\) for example 1

<table>
<thead>
<tr>
<th>Type of space-time interval, (s^2 = \Delta r^2)</th>
<th>SPACE-LIKE</th>
</tr>
</thead>
</table>

It is observed that in S′ the events are no longer simultaneous, the event \(E_1\) occurs after \(E_2\) with \(\Delta t' = 8.02339 \times 10^{-9}\) s (8 splits of second).
Example 2 (Time dilation). Given a punctual process in the fixed inertial reference frame $S$ which lasts a time interval $\Delta t$. We can consider the process like a succession of events in a certain point of specified coordinates in relation to frame $S$. We mark with $E_1$ and $E_2$ the beginning and end events of the process and consider: $E_1$: $x_1 = 1 \text{ m}, y_1 = 2 \text{ m}, z_1 = 0.5 \text{ m}, t_1 = 1 \text{ s}; E_2$: $x_2 = 1 \text{ m}, y_2 = 2 \text{ m}, z_2 = 0.5 \text{ m}, t_2 = 5 \text{ s}$.

Like in the previous examples, the mobile frame $S'$ moves from $S$ with a constant velocity in any given direction so that the vector $\vec{v}$ has the following components $(v_x, v_y, v_z) = (0.90 \text{ c}; 0.10 \text{ c}; 0.20 \text{ c})$.

The comparison between events obtained with the help of the spreadsheet in the two reference frames is given in Tables 4–6:

Table 4

<table>
<thead>
<tr>
<th>Comparison between events in $S$</th>
<th>$\Delta r$ [m]</th>
<th>0</th>
<th>$E_1$ in the same place with $E_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reporting in space</td>
<td></td>
<td></td>
<td>$E_1$ before $E_2$</td>
</tr>
<tr>
<td>Reporting in time</td>
<td>$\Delta t$ [s]</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Space-time interval, $s^2 = \Delta r^2 - c^2 \Delta t^2$ [m$^2$]</td>
<td></td>
<td></td>
<td>$-1.43801 \times 10^{18}$</td>
</tr>
</tbody>
</table>

Table 5

<table>
<thead>
<tr>
<th>Comparison between events in $S'$</th>
<th>$\Delta r'$ [m]</th>
<th>2972116427</th>
<th>$E_1$ in a different place than $E_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reporting in space</td>
<td></td>
<td></td>
<td>$E_1$ before $E_2$</td>
</tr>
<tr>
<td>Reporting in time</td>
<td>$\Delta t'$ [s]</td>
<td>10.69044968</td>
<td></td>
</tr>
<tr>
<td>Space-time interval, $s' = \Delta r'^2 - c^2 \Delta t'^2$ [m$^2$]</td>
<td></td>
<td></td>
<td>$-1.43801 \times 10^{18}$</td>
</tr>
</tbody>
</table>

Table 6

<table>
<thead>
<tr>
<th>Type of space-time interval, $s^2$</th>
<th>TIME-LIKE</th>
</tr>
</thead>
</table>

It can be observed that the time interval of the process in $S'$ is bigger than in $S$ and the two events are in different spatial places in $S'$.

3.2. THE “RELATIVISTIC ADDITION OF VELOCITIES” INSTRUMENT

With the help of this instrument we can compound in a relativistic way the velocities in conformity with the relation (7). Knowing the components of the velocity in a particle in the fixed inertial reference frame $S$ we can determine the components of the velocities of the same particle in the mobile reference frame $S'$. In Fig. 2 we have rendered the spreadsheet associated to the instrument with the sections “Data input” and “Results”.
1. Grigore, C. Miron, E. S. Barna

Fig. 2 – The main spreadsheet of the instrument “The relativistic addition of velocities”. The colored versions could be accessed at http://www.rrp.infim.ro/rrp/.

In the section “Data input” the following data are introduced: speed of light in vacuum, $c$, expressed in km/s, the velocity of the particle in the reference frame $S$ expressed by the ratios between the components of the velocity vector $\vec{v}$ on the axes $O_x$, $O_y$, $O_z$ and the speed of light in vacuum, $\beta_i = \frac{v_i}{c}$, $i = x, y, z$, the velocity of the reference frame $S'$ in relation to $S$ under the form of ratios between the components of the velocity vector $\vec{v}'$ on the axes $O_x$, $O_y$, $O_z$ and the speed of light in vacuum, $\beta_{0i} = \frac{v_{0i}}{c}$, $i = x, y, z$.

In order to transcribe in Excel the vectorial relation (7) to calculate the velocity of the particle in $S'$ knowing the velocity in $S$ and the velocity of $S'$ to $S$ we name the cells in which data are introduced: “Speed_L” for cell D5, “Beta_0x” for cell B12, “Beta_0y” for cell C12 and “Beta_0z” for cell D12.

In the section “Results” we first calculate the quantities: the factor $\beta$, the factor $\beta_0$ and the factor Lorentz $\gamma$. We have:

$$\beta = \sqrt{\beta_x^2 + \beta_y^2 + \beta_z^2}; \quad \beta_0 = \sqrt{\beta_{0x}^2 + \beta_{0y}^2 + \beta_{0z}^2}; \quad \gamma = \left(1 - \beta_0^2\right)^{-1/2}. \quad (18)$$
With the notations we introduced, and the name Beta_0 of the cell in which the result is returned for \( \beta_0 \), the transcription of the relations (18) in Excel becomes:

- **Factor \( \beta \):**
  \[
  \text{IF}(\sqrt{\text{Beta}_x^2 + \text{Beta}_y^2 + \text{Beta}_z^2} < 1; \sqrt{\text{Beta}_x^2 + \text{Beta}_y^2 + \text{Beta}_z^2}; "NO")
  \]

- **Factor \( \beta_0 \):**
  \[
  \text{IF}(\sqrt{\text{Beta}_0x^2 + \text{Beta}_0y^2 + \text{Beta}_0z^2} < 1; \sqrt{\text{Beta}_0x^2 + \text{Beta}_0y^2 + \text{Beta}_0z^2}; "NO")
  \]

- **Factor Lorentz \( \gamma \):**
  \[
  \text{IF}(\text{Beta}_0 < 1; 1/\sqrt{1 - \text{Beta}_0^2}; "NO")
  \]

We express the velocity of the particle in \( S' \) through the ratios \( \beta_i' = v_i'/c \).

Taking into account the vectorial relation (7) we obtain:

\[
\beta_i' = \frac{\beta_i + \gamma^{-1} \left( \sum_{k=1}^{3} \beta_{ik} \beta_{0k} \right) \beta_{0i} - \gamma \beta_{0i}}{\gamma \left( 1 - \sum_{k=1}^{3} \beta_{ik} \beta_{0k} \right)}, \quad (19)
\]

where \( i = x, y, z \).

With the previous notations and naming the cells in which we calculate the factors \( \beta \) and \( \gamma \) with Beta and respectively Gamma, the transcription in Excel of the relation (19) for \( I = x \) leads to:

\[
\text{IF}(\text{AND}(\text{Beta} < 1; \text{Beta}_0 < 1); (\text{Beta}_x + ((\text{Gamma} - 1)/(\text{Beta}_0^2)) \times (\text{Beta}_x \times \text{Beta}_0 + \text{Beta}_y \times \text{Beta}_0y + \text{Beta}_z \times \text{Beta}_0z)) \times \text{Beta}_0x - \text{Gamma} \times \text{Beta}_0x) / (\text{Gamma} \times (1 - (\text{Beta}_x \times \text{Beta}_0x + \text{Beta}_y \times \text{Beta}_0y + \text{Beta}_z \times \text{Beta}_0z))); "NO")
\]

Similarly are written in Excel the formulas for the calculus ratio \( \beta'_y \) and ratio \( \beta'_z \).

We utilized the logical functions IF and AND for the same reasons as in the formula from the instrument “The analysis of events”, namely to return the message “NO” as an error message in the situation in which the user introduces from the keyboard values for the components of the velocity vector \( \vec{v} \) of the particle in \( S \) and, respectively, values for the components of the velocity vector \( \vec{v}' \) of \( S' \) to \( S \) that leads to a module of the velocity of the particle in \( S \), respectively, to a module of the velocity of \( S' \) from \( S \) so that it exceeds the speed of light in vacuum.

Under the table in which the values \( \beta'_x, \beta'_y \) and \( \beta'_z \) are returned a table is rendered in which we make the conversion in km/s for the velocity of \( S' \) from \( S \), the velocity of the particle in \( S \) and the velocity of the particle in \( S' \) for each component according to the three axes of coordinates.

Next to the tables for data input and the ones for results, there is the diagram which compares the components of the velocity of \( S' \) from \( S \) with the components of the velocities of the particle in the two reference frames according to the three axes of coordinates. When changing the data, the diagram is modified correspondingly, offering an overall visual image on the relativistic composition of velocities. As observed in the diagram, each component is rendered with the sign resulted from the calculus, plus or minus, as the projection of the material point on the respective axis moves in the sense of the axis or opposing it.
3.3. THE “LENGTH CONTRACTION” INSTRUMENT

This instrument contains two spreadsheets, namely the main spreadsheet in which results are obtained according to the data input, and a spreadsheet for the source table of the diagram associated to the data from the main sheet.

In the main spreadsheet we can obtain the kinematic length, \( L \), according to the static length, \( L_0 \), for a certain ratio \( \beta = \frac{v}{c} \) utilizing the relation (4), but the inverse problem can be solved, calculating the ratio \( \beta = \frac{v}{c} \) so that the kinematic length takes a certain value \( L \) according to the static length \( L_0 \). Thus, considering the unknown \( \beta \) in the relation (4) we obtain its value according to \( L \) and \( L_0 \):

\[
\beta = \sqrt{1 - \left( \frac{L}{L_0} \right)^2}.
\]  

(20)

The calculations are done in the particular situation in which \( S' \) moves from \( S \) in the direction of the axis Ox.

Thus, the spreadsheet contains the section “Problem \( \beta \rightarrow L \)” and the section “Problem \( L \rightarrow \beta \)”.

In the former, the ratio \( \beta = \frac{v}{c} \) is introduced and the kinematic length, \( L \), is calculated after we introduce a value for the static length \( L_0 \).
In the latter, a value is introduced for the kinematic length and the ratio $\beta = v/c$ is calculated according to the relation (20).

By modifying the data input we can trace the changes in results.

Next to the two sections we have the graphic that renders the dependency of the kinematic length, $L$, on the ratio $\beta = v/c$ at a certain value of the static length, $L_0$. It can be observed how the curve $L = L(\beta)$ tends asymptotically towards zero at $\beta = 1$ (at the speed of light L becomes equal to zero). On the curve we have marked the point that corresponds to the pair of values ($\beta, L$) and it moves on the curve once the value of $\beta$ is modified.

The transposition in Excel of the calculation formulae is made through a procedure similar to those presented in the previous instruments.

**Example.** For $L_0 = 2$ m and $\beta = 0.75$ ($S'$ moves with 75% of the speed of light in vacuum from $S$) we obtain $L/L_0 = 0.661$ (the kinematic length represents 66.10% of the static length), respectively $L = 1.323$ m. For the kinematic length to take the value $L = 1$ m when the static length is still $L_0 = 2$ m we obtain $\beta = 0.866$ (the velocity of $S'$ from $S$ is 86.6% of the speed of light in vacuum).

The main spreadsheet of the instrument is presented in Fig. 3.

### 4. CONCLUSIONS

Spreadsheets can be successfully integrated in Physics lessons. In the process of teaching and learning the special relativity theory, spreadsheets help both teachers and students. Thus, teachers can offer many numerical examples and rapid feedback from students. With a large number of examples and particular situations, students can more easily understand the consequences of the Lorentz transformations and they can overcome the inherent obstacles met when in the study of relativistic Physics. One can verify the variation of quantities and the invariance of others that are specific to relativistic phenomena. Moreover, one can rapidly verify through the data input that at small velocities compared to the one of light in vacuum, the results of the classic kinematics are obtained.

A part of the presented instruments have the advantage that they treat the relative movement of the two reference frames in which the transformation of physical quantities is analyzed in the case in which this movement takes place according to any given direction in relation to the axes of the frames, and not in the particular case of the movement according to a certain axis, case which is presented in the majority of the classic course books on the special relativity theory.

We can say that spreadsheets facilitate a variety of student-centered learning styles characterized by opening, constructivism and investigations [19].

The teachers can use adequate strategies and informatics tools to introduce them to their science interested students, as is related in many physics education papers [23–26].
Acknowledgments: This work was supported by the strategic grant POSDRU/159/1.5/S/137750, "Project Doctoral and Postdoctoral programs support for increased competitiveness in Exact Sciences research" cofinanced by the European Social Found within the Sectorial Operational Program Human Resources Development 2007–2013.

REFERENCES