Romanian Reports in Physics, Vol. 65, No. 4, P. 1281–1300, 2013

CLUSTERIZATION PROBABILITY IN ALPHA-DECAY ^{212}Po NUCLEUS WITHIN CLUSTER-FORMATION MODEL; A NEW APPROACH

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Received June 12, 2012

Abstract. A clusterization theory is presented to describe the clusterization probability (preformation factor). The clusterization states are described as a quantum-mechanical cluster-formation state in a proposed cluster-formation model to determine the preformation factor of alpha-decay process in radioactive nuclei. The formation of alpha particle inside the parent nuclei is considered within two postulates; the compound nucleus of Bohr's assumption and the surface effect. The total and formation energy are obtained from the from the binding energies differences. This model is tested for ^{212}Po for the alpha-cluster decay. The calculated preformation factor 0.54 has shown good agreements with that of some others as reported. As such, this model could give more insight to the understanding of the nuclear structure in the radioactive nuclei.

Key words: alpha-cluster decay, clusterization probability.

1. INTRODUCTION

One of the most important processes that have been used to study the nuclear structure of heavy nuclei is the alpha-decay process because of its domination, high-accuracy experimental measurements, and the availability of its microscopic theory. This process spontaneously occurs in most heavy and super-heavy nuclei and is one of the most dominant modes. The simplicity of this process is due to the small number of observables, the decay width, the energy of emitted alpha particle, and some properties of its transition from ground state to another ground state with the same parity. These reasons have pushed researchers to microscopically modulated the shell closure, and stability of nuclei that formed islands of stability, around Z ~ 114 and N ~ 184, by different theories and models [1–10].

Theoretically, Gamow and Condon and Gurney (1928) were the first to present an explanation of α-decay as a quantum tunneling effect. Usually, the

α-decay process is described as a preformed α-particle tunneling through a potential barrier between the cluster and the daughter nucleus. The penetrability or the penetration probability through the barrier can be determined by the well known Wentzel-Kramers-Brillouin (WKB) semiclassical approximation. The absolute α-decay width is mainly determined by R-matrix [11–15], the general formula [16-17], or by direct product of the preformation factor and the penetration probability calculated by using WKB approximation [6,18–25]. In the R-matrix formula, the decay width is a product of the preformation factor (formation amplitude at coulomb radius r_c) of the alpha cluster inside the parent nucleus and the α-cluster penetration probability. However, in the general formula the compound nuclei are considered in resonance within quasi-bound states and the alpha-decay process is a transition from a state represented by a parent wavefunction governed by its Hamiltonian to a state of a daughter with the alpha particle represented by a wavefunction governed by another Hamiltonian. The preformation factor is sometimes included to be multiplied by the decay width [16, 17]. The preformation factor (cluster formation probability and also presented in term of the spectroscopic factor) is defined as the quantum-mechanical probability of finding the cluster inside the parent nucleus [4-5, 21, 26]. The preformation factor is very important because it reflects information about the nuclear structure, since it is a good indicator of deformation in the nuclei. This is explained as the high probability of alpha-cluster formation from the last-shells nucleons that leads to deform the sphericity of the nuclear surface [6].

With the use of R-matrix to calculate the alpha-decay width of ^{212}Po, Tonozuka and Arima (1979) [11] calculated the formation amplitude alpha cluster in ^{212}Po using high configuration mixing up to 13 $\hbar\omega$ bases of harmonic-oscillator shell model. The improved calculation was still short of the experimental width by a factor of 23. Dodig-Crnkovic *et al.* (1985,1989) [12–13] recalculated and improved the formation amplitude using the multistep shell model method in which the interaction among the valence nucleons of ^{212}Po were included in different considerations. When high-lying states were considered with some pairing interactions, the alpha-decay width was less than the experimental by a factor of 10 [12]. In an attempt to reduce the dimension of matrix elements (using surface delta interaction within truncated model space), the calculation led to enhance the alpha-decay width to be less from the experimental data by a factor of several times[13]. Varga *et al.* (1992) [14–15] combined the shell model with cluster model within bases of 538 dimensions and included the interaction among the four nucleons within the model space up to the next magic number for ^{208}Pb. They confirmed the existence of alpha-core structure. Varga and Lovas (1994) repeated the work of Varga *et al.* (1992) [14–15] but with Gaussain-bases shell model to reduce the dimension of the bases. They included 400 bases and 120 configurations. The

results of alpha-decay width were about same as that of Varga *et al.* (1992) in which 631 bases and 193 configurations were included to get the best agreement between the calculated and experimental binding energy.

For groups of heavy nuclei, the general form and WKB approximation were adopted to reproduce the alpha-decay widths or half-lives by using different alpha-core potential to provide more understanding of the nuclear structure. In some calculations the WKB approximation was adopted, missing the effect of the cluster formation by assuming a value of 1 for the preformation factor when the alpha-decay width were calculated for even-even nuclei and for ^{212}Po [27] but a value less than one for odd-even, even-odd, or odd-odd [18–20, 27]. In other calculations the preformation factor was assumed as a quantum-mechanical probability with a value less than one and different for each nucleus. This value could easily be extracted by dividing the experimental value of the alpha-decay rate by the penetration factor of the nucleus. This led to a phenomenological formula that could be used to determine the preformation factor for each nucleus. This assumption improved the reproduction of alpha-decay widths [28], but, did not add any more understanding to the microscopic theory of the models used in this process since the potentials of the nuclear structure had contributions only in the penetration factors. The parameters of the suggested potential were mostly determined by fitting to the experimental values of alpha-particle energy, therefore each different potential can produce different preformation factors. In actual fact, the preformation factor has neither been calculated for each nucleus by using the current models of nuclear structure, nor its realistic value have been determined [1, 4–6, 21, 23, 25, 29–31].

Recently, Qian and Ren (2011) [24] used a Z-dependent formula based on two-level model and obtained the value of ^{210}Po preformation factor. This formula is actually similar to the phenomenological formula in which the spectroscopic factor is only the chosen parameter. In the two-level model [32–33] the pairing interactions are considered for proton-proton, neutron-neutron, and proton-neutron with the pairing energies taken from the separation energies of the nucleons.

In our work we present a new quantum-mechanical theory for the clusterization effect. The clusterization effect is interpreted as a formation of clusters inside a nucleus. Any type of cluster is considered as a quantum-mechanical cluster-formation state. The eigenvalues, formation energies, of these states can be calculated from the binding energies. The formation or preformation factor is derived by using Schrödinger equation. This model in its simplest form is tested for ^{212}Po nucleus in which only alpha-cluster is considered. In section (2) the theory of clusterization and the cluster-formation model is presented to derive the preformation factor. An application for the theory is presented for alpha decay of ^{212}Po in section (3), the method of formation energy calculation for alpha cluster and results are presented in section (4), and the discussion for ^{212}Po are given in section (5).

2. THEORY OF CLUSTERIZATION AND CLUSTER-FORMATION MODEL

When a system exhibits a behavior of more than one cluster, there is more than one state of clusterization. The total wave function of the system is a linear combination of these clusterization states. These states have same energy and angular momenta but different probability densities. Each clusterization state is due to the different Hamiltonian for the system. If the system of A nucleons is considered as one cluster, its Hamiltonian operator in the lab system is

$$H_0(\vec{r}) = \sum_{i=1}^{A} \left(\frac{\hat{p}_i^{\,2}}{2m_i} + \sum_{\substack{j=1 \\ (j>i)}}^{A} V_{ij} \right), \tag{1}$$

where $\hat{p}_i$ and m_i are the momentum operator and the mass of the i^{th} nucleon which interacts with each j^{th} nucleon in the system by a two-body potential V_{ij}. In the second summation notation the parentheses *(j > i)*, are used to show that this summation is only for j index. This Hamiltonian is invariant to the permutations of position vectors $\vec{r}_i$. When this Hamiltonian is set to the Time-Independent Schrödinger Equation (TISE) as

$$H_0 \Psi_0(\vec{r}) = E \Psi_0(\vec{r}), \tag{2}$$

where Ψ_0 is the normalized wavefunction of the system. If the system is considered in two groups with A_d nucleons and A_c nucleons, then

$$A = A_d + A_c, \tag{3}$$

where A_d are the nucleons of i = 1 to A_d and A_c are the nucleons of i = (A_d+1) to A. Then the Hamiltonian can be written as

$$H_1 = \sum_{i=1}^{A_d} \left(\frac{\hat{p}_i^{\,2}}{2m_i} + \sum_{\substack{j=1 \\ (j>i)}}^{A} V_{ij} \right) + \sum_{i=A_d+1}^{A} \left(\frac{\hat{p}_i^{\,2}}{2m_i} + \sum_{\substack{j=1 \\ (j>i)}}^{A} V_{ij} \right). \tag{4}$$

The two outer summations are for two groups of nucleons but the potential energy terms are expanded to all nucleons. To separate this interaction it is possible to consider the kinetic energy of any nucleon as a sum of two parts, $K_i = K_{oi} + K_{1i}$; the first is due to the interaction of i^{th} nucleon with the other nucleons in its group and the second is for the nucleons in the other group. In terms of operators,

$$\hat{p}_i^{\,2} = \hat{p}_{oi}^{\,2} + \hat{p}_{1i}^{\,2}. \tag{5}$$

Substituting Eq.(5) in Eq.(4) and splitting the potential-energy terms of the other group from each sum and rearrange them, we obtain Hamiltonian for a certain value of A_d ,

$$H_1(\vec{u}) = \sum_{i=1}^{A_d}\left(\frac{\hat{p}_{oi}^{\,2}}{2m_i} + \sum_{\substack{j=1 \\ (j>i)}}^{A_d} V_{ij}\right) + \sum_{i=A_d+1}^{A}\left(\frac{\hat{p}_{oi}^{\,2}}{2m_i} + \sum_{\substack{j=A_d+1 \\ (j>i)}}^{A} V_{ij}\right) + \sum_{i=1}^{A_d}\left[\frac{\hat{p}_{1i}^{\,2}}{2m_i} + \sum_{j=A_d+1}^{A}\left(\frac{\hat{p}_{1j}^{\,2}}{2m_j} + V_{ij}\right)\right]. \quad (6)$$

The use of the different space coordinates $\vec{u}$ is due to the redistribution of nucleons in two groups. When H_1 is set in TISE as

$$H_1\Psi_1(\vec{u}) = E\Psi_1(\vec{u}) . \quad (7)$$

Ψ_1 is obtained to be the normalized wavefunction of the system when it behaves as a two-cluster system. So it is to describe one of the clusterization states. In Eq.(6), it is possible to consider different values of A_d to obtain different Hamiltonians H_2, H_3, .., or this equation can be rearranged to be written for three groups (three-cluster system) to obtain more different Hamiltonians. Then, the total wavefunction is a linear combination of these different clusterization states as as

$$\Psi = a_0\Psi_0 + a_1\Psi_1 + a_2\Psi_2 + ... + a_n\Psi_n . \quad (8)$$

These wavefunction are a complete set of n coordinate space. Each wavefunction of clusterization state is defined with different space coordinate, so, any is orthogonal to the other, and found from the solution of TISE;

$$H_i\Psi_i = E\Psi_i, \quad i = 0,1,...,n . \quad (9)$$

The constants (a_0, a_1, .., a_n) are the amplitudes of each the wavefunctions states and subjected to

$$|a_o|^2 + |a_1|^2 + .. + |a_n|^2 = 1 . \quad (10)$$

The total Hamiltonian of the system is

$$H = H_0 + H_1 + .. + H_n \quad (11)$$

which should be linear for the eigenfunction of Eq.(8) as

$$H\Psi = E\Psi . \quad (12)$$

The linearity can be set as long as the states are orthogonal and the Hamiltonian of each state is with different space coordinate. The expectation value of energy E from the total wavefunction, in Dirac notation, is

$$E = \langle\Psi|H|\Psi\rangle . \quad (13)$$

when Eq.(8) and Eq.(11) are substituted in Eq.(13) we obtain

$$E = |a_o|^2 E + |a_1|^2 E + ... + |a_n|^2 E . \quad (14)$$

The probability of finding the system in the clusterization state Ψ_o is $|a_o|^2$ and in the clusterization state Ψ_1 is $|a_1|^2$. There is a probability for each clusterization state which is energetically favored, *i.e.* there should be implicitly specific energy E_f responsible for each clusterization state, so Eq.(14) can be written as

$$E = E_{f_0} + E_{f_1} + .. + E_{f_n} . \quad (15)$$

Then the probability of i^{th} clusterization state or the preformation factor (clusterization probability) P_i can be written as

$$|a_i| = P_i = \frac{E_{f_i}}{E} . \quad (16)$$

From Eq.(16), the clusterization-state probability depends on the total energy of the system and another energy which is desired to be determined within a model that well describes the clusterization effect in the structure of the system.

The inclusion of clusterization effect in the structure of the nuclear system within a model requires the considerations of many clusterization states: the system is one cluster, the system is made of two clusters, made of another two different clusters, three clusters, etc. Magic-number light and medium nuclei in ground states are examples of the one-cluster system. Radioactive nuclei that emit clusters are examples of one and two-cluster nuclei (more than cluster may be called sub clusters or sibling clusters).

For any one-cluster (or mono-cluster) system, the cluster is formed due to high binding energy among the nucleons; hence there is only one clusterization state. The energy responsible for this one cluster formation is the total energy of the system, so, in accordance to Eq.(16), the probability of this state is one. For the a two-cluster (di-cluster) system, Eq.(6) can be considered, for a certain value of A_d, to determine the total energy of the system and the energy responsible for its clusterization. This equation contains three main sums. In each one there are momentum operators and potential energy written in different space coordinates ($\vec{u}$: ξ_i, η_i and ρ_i), which could be substituted in TISE to obtain the quantum-mechanical states and the wavefunction that describe the nucleons, so these three terms can be written in term of different Hamiltonians as

$$H = H_{A_d}(\xi) + H_{A_c}(\eta) + H_{A_d \otimes A_c}(\rho). \quad (17)$$

The first Hamiltonian $H_{A_d}(\xi)$ is for the first cluster and will be replaced by H_{f_1}, and the second is for the second cluster and will be replaced by H_{f_2}. However the

third is for relative motion between the two clusters and will be replaced by H_r. Then Eq.(17) and its corresponding energy equation can be rewritten as

$$H = H_{f_1} + H_{f_2} + H_r, \tag{18}$$

$$E = E_{f_1} + E_{f_2} + E_r. \tag{19}$$

The Hamiltonian of Eq.(18) can be written as a sum of two $H = H_f + H_r$; one for the clusters formation ($H_f = H_{f_1} + H_{f_2}$) and the other for the interaction between the two clusters. Writing them in TISE, we obtain

$$H\Phi = (H_f + H_r)\Phi. \tag{20}$$

This equation can be separated in accordance to the Hamiltonians of Eq.(17) and the total wavefunction Φ of the system can be expanded as

$$\Phi = \Phi_{f_1}(\xi)\,\Phi_{f_2}(\eta)\,\Phi_r(\rho) \tag{21}$$

The three wavefunctions are supposed to be defined on their space coordinates ξ, η and ρ, and found from the solution of TISEs;

$$H_{f_1}\Phi_{f_1}(\xi) = E_{f_1}\Phi_{f_1}(\xi), \tag{22}$$

$$H_{f_2}\,\Phi_{f_2}(\eta) = E_{f_2}\,\Phi_{f_2}(\eta), \tag{23}$$

$$H_r\Phi_r(\rho) = E_r\Phi_r(\rho). \tag{24}$$

These wavefunctions are eigenfunctions to the operators; H_{f_1}, H_{f_2} and H_r with eigenvalues E_{f_1}, E_{f_2}, and E_r respectively. The first two Eqs.(22, 23) are related to the cluster-formation Hamiltonian $H_f = H_{f_1} + H_{f_2}$, so, Eq.(20) can be

$$H\Phi = (E_{f_1} + E_{f_2} + E_r)\Phi = (E_f + E_r)\Phi = E\Phi. \tag{25}$$

The total energy E is the eigenvalue of the Hamiltonian of Eq.(17). The system described in the wavefunction Φ in Eq.(25) is considered to be a two-cluster system, *i.e.* Eq.(23) is only to describe one clusterization state. There are three energies in Eq.(25): E_f, E_r, and E. To determine the amount of energy that is responsible for the cluster formation to be set in Eq.(16), it is important to mention that the agglomeration of a part of nucleons of the system to a cluster occurs when the agglomerated nucleons are bound enough to each other and not bound enough to the other nucleons. This mean, in terms of total binding energy, the cluster formation occurs when the total binding energy of the cluster E_f is large and its binding energy E_r to the others is small. In addition, the clusterization increases when the total energy of the system E is large because clusterization effect is so much observable in heavy nuclei. Since, the energy responsible for the

cluster formation is directly proportional to the absolute values of E_f and E, and inversely to E_r, we assume that this energy is E_f because $E_f = E - E_r$ (see Eq.(25)). As a result, Eq.(16) can be rewritten for the clusterization state of Eq.(25) as

$$P_{\Phi} = \frac{E_f}{E}. \tag{26}$$

The preformation factor of the cluster formation probability P_{Φ} can theoretically be calculated as

$$P_{\Phi} = \frac{\langle \Phi | H_f | \Phi \rangle}{\langle \Phi | H | \Phi \rangle}. \tag{27}$$

When the cluster formation wavefunction is difficult to obtain, it is possible to use $E_f = E - E_r$ which can be written in terms of the expectation values for the calculation of P_{Φ} as

$$P_{\Phi} = \frac{\langle \Phi | H | \Phi \rangle - \langle \Phi | H_r | \Phi \rangle}{\langle \Phi | H | \Phi \rangle}. \tag{28}$$

The derivation of the cluster-formation probability or the preformation factor of a cluster from Eq.(27) or Eq.(28) is based on the clusterization theory that enables to write the total wavefunction in more generality and based on presenting the cluster-formation model for the structure of nuclear system.

For the clusterization state described in Eq.(21) the system is considered in the clusterization state of two clusters. Each cluster is described by the cluster-formation eigenfunctions: Φ_{f_1}, Φ_{f_2}, as in Eqs.(22, 23), and normalized on its sub space. These two wavefunctions are the cluster formation states and should be considered when the system exhibits the clusterization effect especially when the preformation factor or the cluster-formation probability is required to be calculated from Eq.(27). The cluster-formation wavefunction is always referred in works as an intrinsic wavefunction.

3. PREFORMATION FACTOR IN ALPHA DECAY

One of the approaches of the alpha-decay process is the preformation of alpha cluster inside the parent nucleus by a certain interaction which is in charge of only this preformation. In compound nuclei, there may be many possibilities of formations of different clusters [34], but for simplicity, we assume that there are two possible clusterization states; one Ψ_{α} is for two cluster: the alpha-cluster formation state and the daughter-cluster formation state, and the second Ψ_0 is for

all other possible cluster-formation states. The total clusterization wave function of the parent nucleus Ψ can be written as

$$\Psi = a_0 \Psi_0 + a_\alpha \Psi_\alpha , \tag{29}$$

where a_0 is the amplitude of the parent clusterization state without the alpha-formation (or alpha clusterization), and a_α is the amplitude of the clusterization state of the alpha-cluster formation state. The cluster formation inside the parent nucleus was potentially studied with the alpha decay and emphasized in literature review [35], and the clustering effect played a significant role in the improvement of the calculations of alpha-decay widths [4–5, 21, 28, 35].

The formation Hamiltonian operator H_f for these states consists of the potential responsible for the formation of the clusters. In many-body system, it is so difficult to get a form for this potential and any other nuclear potential from the available nuclear model [4–5, 21]. In the preformed cluster model the cluster states are derived from the fragmentation theory in which the cluster states are functions of dynamical collective coordinates of mass and charge asymmetries [2, 26].

The cluster-formation probability of alpha cluster (as in Eq.(27)), P_α is defined as the probability of finding the formed-alpha particle inside a nucleus with the daughter, as

$$P_\alpha = \frac{\langle \Psi_\alpha | H_f | \Psi_\alpha \rangle}{\langle \Psi | H | \Psi \rangle} . \tag{30}$$

This equation can also be derived using a projection operator $\hat{P}_\alpha$ for alpha-daughter cluster-formation as

$$\hat{P}_\alpha = | \Psi_\alpha \rangle \langle \Psi_\alpha | , \tag{31}$$

with ψ_α is the normalized wavefunction of alpha-cluster formation. Lovas *et al.* (1998) [28] defined the preformation factor as a projection of the parent nuclei wavefunction on the alpha-cluster wavefunction. Applying this to the total clusterization wavefunction, we again obtain the preformation factor,

$$P_\alpha = \langle \Psi | \hat{P}_\alpha \ | \Psi \rangle = \langle \Psi | \Psi_\alpha \rangle \langle \Psi_\alpha | \Psi \rangle = |a_\alpha|^2 , \tag{32}$$

which is equivalent to Eq.(16).

4. CALCULATIONS AND RESULTS

To calculate the alpha-formation probability theoretically we need to find the total wavefunction and the cluster-formation (intrinsic) wavefunction of the

clustering alpha for Eq.(30). These wavefunctions are supposed to be determined from the solution of TISE in Eq.(20) and Eqs.(22, 23). The determination of the wavefunctions are not our target in this work although these are required in Eq.(30). The target of the present work is the evaluation of the cluster-formation model by the determination of two values of energy; the total energy of the parent nucleus and the formation energy of the alpha cluster. As long as it is possible to use the experimental value of these two energies it is better to investigate the validity of this approach and the contribution of each to the alpha-cluster formation probability.

4.1. FORMATION-CLUSTER STATE ENERGY OF ^{212}Po

The formation energy of a cluster-formation state is the total intrinsic energy of the cluster and it is equivalent to the total energy of a cluster when the cluster is at rest. This energy is responsible for the cluster formation in any system from its constituents. When the cluster is considered formed from some of the system nucleons, its cluster formation energy is equivalent to the sum of the binding energies among the cluster nucleons, or it is the subtraction of the interaction energy between the cluster nucleons and the other nucleons from the sum of the cluster-nucleons energies. The experimental formation energy of alpha cluster (E_f) can easily be obtained from the calculations of the binding energy differences, the mass defect between the alpha particle and the four free nucleons. It can found as follows;

$$E_f = E_{\alpha d} - Q_\alpha, \tag{33}$$

where $E_{\alpha d}$ is the alpha decay energy given as

$$E_{\alpha d} = B(A,Z) - B(A-4,Z-2), \tag{34}$$

and Q_α is equivalent to the separation energy of alpha particle, and is defined as

$$Q_\alpha = B(A-4,Z-2) + B(4,2) - B(A,Z). \tag{35}$$

The Q-values and the alpha-decay energy were used to extract the alpha-decay preformation factor [36]. The second energy that contributes to the cluster-formation probability of Eq.(30) is the total energy E for the parent nucleus. It can also be found from the binding energy differences, so Eq.(30) can written as

$$P_\alpha = \frac{E_f}{E}. \tag{36}$$

The theory mentioned above is presented on the basic of the possibility of existence an interaction between any two nucleons inside the nucleus. Therefore, two assumptions are adopted to determine this energy. One is Bohr's assumption of compound nucleus [37] which leads a proposal of this energy as the total energy of the whole parent nucleus, therefore the total binding energy of the parent nucleus is

$$E = E_{Bohr} = B(A, Z), \tag{37}$$

where *B(A,Z)* is the binding energy of a nucleus of atomic mass number *A* and atomic number *Z*.

Secondly, for nuclei that undergo the alpha-decay process, the emitted alpha is from the nuclear surface [11, 34, 38–40] . To consider the energy on the surface, the considered system is not the whole parent nucleus but it is for only the surface nucleons. So the total wavefunction Ψ in Eq.(29) is for the surface nucleons. For ^{212}Po, there are four nucleons above the shell closure. Then, in accordance to the surface assumption, the surface-nucleons energy $E_{surf.}$ is the energy responsible to form the alpha cluster and the alpha emission. This energy is equal to the alpha decay energy $E_{\alpha d}$ given in Eq.(34) , or

$$E = E_{surf.} = E_{\alpha d} . \tag{38}$$

4.2. FORMATION PROBABILITY OF ALPHA IN ^{212}Po NUCLEUS

The calculations of the formation energies of ^{212}Po nucleus were done using experimental data from Ref. [41]. The total energy (*E*) of the considered system have been found using Eq.(37) for Bohr assumption of and Eq.(38) for the surface effect in which the total wavefunction is considered without the part of the core (the daughter intrinsic wavefunction). For each calculation, the formation-cluster probability has been found from Eq.(22). The results are shown in Table 1. The major difference between two calculated values of cluster formation P_α is the significant differences in the considerations system. The first value of P_α is for two formation of clusters (two cluster-formation states) whereas in the second the value is for only the alpha-cluster state. The large value of $P_\alpha = 0.994$ based on Bohr assumption reflects the role of the double-shell closure in the daughter of ^{212}Po, which gives more chance to the alpha clustering for the alpha decay. The value of $P_\alpha = 0.54$ based on the surface effect is smaller because it is for the alpha-cluster only and it is relatively large because it is the probability from all possibilities of clustering in only the last four nucleons within the interaction from the daughter.

Table 1

The experimental total energy of the parent ^{212}Po nucleus E , alpha-cluster formation energy E_f , cluster formation probabilities P_α

Basic of the method	E (keV)	E_f (keV)	$P_\alpha = E_f / E$
Bohr assumption	E_{Bohr}=1655771.50	164817.37	0.994
Surface effect	$E_{surf.}$=19341.54	10387.38	0.54

5. DISCUSSION

The formation of any cluster inside the nucleus is proposed as a quantum-mechanical formation-cluster state with its eigenvalues of formation energy is calculated from the measured binding energies of nuclei and energy separation. ^{212}Po nucleus was chosen to test this postulate because this typical nucleus consists of two protons and two neutrons in its model space after the shell closure. Calculations for this nucleus were done within R-matrix theory of decay by others to test the shell model, the hybrid model (shell and cluster model), and the Bardeen, Cooper, and Schrieffer (BCS) model. Within some of these models the alpha-decay energy and the ground state binding energy were reproduced to test the type of two-nucleon force, and the alpha-cluster formation was presented in different forms depending on the alpha-decay theory used. In addition, the preformation factor of ^{212}Po was also extracted by others who reproduced the experimental alpha-decay widths of certain groups of nuclei using WKB approximation and the general form of decay width.

5.1. FORMATION ENERGY OF ALPHA CLUSTERIZATION IN ^{212}Po

The calculated formation energy of alpha E_f for ^{212}Po (as given in Table 1) is 10 387.38 keV. The value indicates a bound system, the alpha cluster. Comparison with the formation energy of the free alpha ($B_\alpha = 28.295$ MeV the total binding energy of alpha particle), indicates that the alpha cluster is already preformed with all four nucleons but its intrinsic energy is less than that of the free alpha points to two effects; the volume expansion and the transient effect. Considering the *r*-dependence of the nuclear force it will be easy to deduce that there is a volume expansion in the interior alpha with the nucleon density less than that of the exterior alpha. Brink and Castro (1973) [38] studied the stability of nuclei through the density of the nuclear matter due to different nucleon-nucleon forces and found that the alpha cluster is formed when the density of the nucleon becomes one third the density of nuclear center density. Considering the alpha-

decay process in ^{212}Po and the differences between the interior and exterior alpha cluster indicates that there is a transient effect terminates with exterior alpha particle with less volume. This transient property of alpha-cluster formation inside the parent nucleus was reported by Hodgsona and Betak (2003) [34]. This formation energy could be a very good indicator to clusterization that could happen inside nuclei.

5.2. ALPHA-CLUSTER FORMATION PROBABILITY OF ^{212}Po

There is no experimental preformation factor for comparison and any estimated values of preformation were either extracted from fitting experimental data of alpha-decay width within a model with some adjustable parameters or determined by adopting the shell and the cluster models within complicated calculations. Comparison of the preformation factor of our approach with those of others that use microscopic theories and methods to determine a specific preformation factors for each nucleus was also conducted. These methods included the use of microscopic models for the structure of the parent nuclei. There were many attempts, as in Table 2, to calculate the preformation factor of ^{212}Po in the eighties and nineties. All these attempts used the shell-model configurations for ^{212}Po as a core of ^{208}Pb+4 nucleons, and used the R-matrix formula in which the formation amplitude is a function of radial distance, and the value of preformation factor was found at the surface where the radius is called the Coulomb radius r_c, about 8.2 fm for ^{208}Pb. The calculation of the proposed model in our present work is based on experimental data and free from parameter which may lead to realistic values of preformation factors.

As in Table 2, Tonozuka and Arima (1979) [11] used the R-matrix to calculate the formation amplitude of alpha cluster in ^{212}Po using high configuration mixing up to 13 $\hbar\omega$ bases of harmonic-oscillator shell model neglecting the clusterization process through the exclusion of the nucleon-nucleon interactions. The improved calculation was still less than the experimental width by a factor of 23. The value of the formation amplitude was found 0.00013 at 9.6 fm of Coulomb radius. In our approach the clusterization is the main effect and the energy required for this can microscopically be described as the sum of all nucleon-nucleon interactions among the nucleons that form the alpha cluster. When these interactions were partially considered using the multistep shell model method in the interaction among the valence nucleons of ^{212}Po the formation amplitude was improved [12-13]. High-lying states with some pairing interactions were considered but the alpha-decay width was found to be less than the experimental by a factor of 11 while the formation amplitude was 0.74 at the Coulomb radius r_c = 8.2 fm [12]. In the truncated model space using surface delta interaction the calculation led to enhance the alpha-decay width to be less from the experimental data by a factor of several times [13]. Varga *et al.* (1992) [14–15] used the R-matrix in combination of

the shell model with cluster model within bases of 538 dimensions and included the interaction among the four nucleons up to the next magic number ^{208}Pb. The Volkov (V1) force and the Brink-Boeker B1 force were tested and found the clustering amount at the same coulomb radius to be 0.23–0.3 confirming the existence of alpha-core structure. The V1 and B1 effective interactions were chosen to be suitable to determine the binding energies of the last four nucleons using the shell model bases for the single particle and to reproduce the alpha-decay energy. Similar work was conducted by Varga and Liotta (1994) [42] but with Gaussian-bases shell model to reduce the dimension of the bases. Bases of 400 and 120 configurations were included. The formation amplitude at about r_c = 8 fm was about 0.1 fm $^{-1/2}$ but their results of alpha-decay width were about same of Varga *et al.* (1992). Bases of 631 and 193 configurations were needed to get the best agreement between the calculated and experimental binding energy. Varga et al. (1992 [15], 1994 [42]) achieved good results when they reproduced the alpha-decay width with a difference from the experimental by a factor less than one. This combination of the two models reflects very good description on the effect of clusterization expressed in clustering amount of about 0.2–0.3. The Bardeen, Cooper, and Schrieffer (BCS) method was also used to reproduce the alpha-decay width which was two-third the experimental value [28]. The model used in the present work adopted the binding energy differences as eigen energies for the cluster states to give direct contribution to the more realistic value of preformation factor.

Table 2

The amount of alpha-clustering in ^{212}Po represented in different forms and expressions and calculated by the use of R-matrix and microscopic theories

Ref.	Expression of clustering	Amount of clustering	Notice	$\Gamma_{exp.}/\Gamma_{cal.\approx}$
Tonozuka and Arima (1979) [11]	Dimensionless Reduced width	0.00013	at Coulomb radius r_c of 9.6 fm	23
Dodig-Crnkovic et al. (1985) [12]	formation amplitude (or reduced width)	0.74 0.11	r_c = 8.2 fm r_c = 9 fm	10
Varga et al. (1992) [14–15]	clustering amount	0.23-0.3	r_c = 8.2 fm	2/3-1
Varga and Liotta (1994) [42]	formation amplitude	0.1	About r_c=8 fm	1/2
Our work	Cluster formation probability	0.54	Total energy from Eq.(38)	-
		0.994	Total energy from Eq.(37)	

According to the proposed model two values of alpha-cluster formation probabilities were calculated (Table 2) to indicate to the contribution of the parent-nucleus total energy (Table 1). Comparing the large value of P_α=0.994 by

Bohr assumption with all previous calculations (Table 2 and 3) has shown that the probability of alpha clustering is great but still smaller than one, and smaller than that of Dodig-Crnkovic *et al.* (1985) [12], 0.74 at small Coulomb radius 8.2 fm. The small preformation factor of Tonozuka and Arima (1979) was due to ignoring the interaction among the nucleons that form the alpha cluster. In the present work, the large value of the preformation factor is due to the inclusion of the interaction among these nucleons and the total energy of the parent nucleus based on Bohr assumption which shows the contribution to both alpha and the daughter clusterization.

When the surface nucleons interactions were considered in the determination of the total energy of the parent nucleus the value of preformation factor was more realistic where $P_\alpha = 0.57$ is within the values (0.1–0.74) as given in Table 2, except that for Tonozuka and Arima (1979) [11], which include the contribution from the surface effect. The small differences may be due to the contrasts of the surface-nucleons-interactions contributions.

Table 3

The preformation factor P_α of alpha-decay in ^{212}Po calculated by different methods

Reference	P_α	Theory of decay	Method of extraction
Buck et al. (1993) [18]	1.00	WKB	Assumed as a large value for even-even nuclei.
Buck et al. (1994) [27]	1.00	WKB	Assumed as a large value for even-even nuclei.
Hoyler et al. (1994) [23]	0.035	WKB	Adjusted and chosen to reproduce the experimental alpha-decay width.
Ni and Ren (2009) [17]	0.56	The general form	From fitting to experimental width to even-even nuclei with $126 < N \leq 176$.
Routray et al. (2009)	1.00	WKB	Assumed as a large value for even-even nuclei.
Our work	0.54	Cluster-formation model	Cluster formation for alpha.
	0.994		Cluster formation for alpha and the daughter.

Many researchers potentially reproduced the alpha-decay widths or half-lives for different groups of radioactive nuclei to provide unified microscopic insight to the nuclear structure of these nuclei. The consideration of alpha-core system yielded a remarkable reproduction of the decay widths, within a deviation factor of a value less than 3. In such calculations (Table 3) the preformation factor was included because of uncertain formation of alpha-core system especially in the initial state of the parent nucleus. The calculated preformation factor completely depended on the calculation of the penetrability, the WKB approximation, except for the work of Ni and Ren (2009) [17] because the general form of decay width was used. All the calculations were performed by adopting the Coulomb potential and a nuclear potential for a system of two-body, alpha-core system, but the

various nuclear potentials used in the calculations of alpha-decay width do not have large differences in the value of the preformation factor [4]. The importance of such calculations is due to the consideration of alpha-core system because within such assumption the interaction of nucleons that form the alpha cluster is already mostly included. Thus it is worth to compare the results of these works with the preformation factor for ^{212}Po (Table 3) in the present work.

Buck *et al.* (1993, 1994) [18, 27] and Routray *et al.* (2009) [30] calculated the alpha-decay width for alpha-decay nuclei. The preformation factor was assumed to be one, the largest for the even-even nuclei and showed that cluster model was good in reproducing the experimental alpha-decay width with a small difference. However, for other nuclei, the preformation factors extracted were less than one, the existence of alpha-core system before the emission of alpha particle was strongly confirmed. Hoyler *et al.* (1994) [23] used the double density folding potential that was relevant for the elastic and inelastic alpha scattering for the alpha-core system with large volume integral and relatively high global quantum number G of alpha oscillatory. The fitting to the experimental half-life of alpha decay led to a small preformation factor of 0.03. The remarkable attempts aim to get good values of preformation factor (Table 2) with Ni and Ren (2009) [17] showed a notable value of 0.54 for the preformation factor through a fitting for only ^{212}Po (Table 3). In addition, Ni and Ren (2009) used the general formal instead of WKB method and the potential was the Wood-Saxon interaction.

The preformation factor of 0.54 calculated (as shown in Table 3) is in a very good agreement with that of Ni and Ren (2009) [17]. A similar value for ^{218}U preformation factor (about 0.3) was obtained by Qian and Ren (2011) [24] when a formula based on two-level model was used. This formula is a parameter and Z dependent, which is actually similar to the phenomenological formula in which the spectroscopic factor is the only chosen parameter. In the two-level model [32–33] the interactions of pairing are considered for proton-proton, neutron-neutron, and proton-neutron. These pairing energies are taken from the separation energies of the nucleons.

As a general noticeable remark in Table 2 and 3, one can consider the value of the preformation factor of alpha decay in ^{212}Po is between $0 < P_\alpha \leq 0.7$. Our calculated preformation of 0.54 is in agreement with that of 0.56 recently determined by Ni and Ren (2009) [17].

6. CONCLUSION

The cluster-formation model proposed in the present work is a good step in finding a realistic value of the preformation factor for nucleus, but further works need to be conducted to confirm its validity for all nuclei. The value obtained also has shown the particle existence of alpha cluster, which can be used for prediction of cluster radioactivity.

The magnitude of the formation energy is a reflection to the cluster-volume expansion before the penetration process. This may be taken into account when the penetrability is calculated. In addition, this magnitude indicates to the deformation on the surface of the parent nucleus.

Acknowlodgements. The authors would like to acknowledge the financial support from the Ministry of Higher Education Malaysia (through the Fundamental Research Grant Scheme) UKM-ST-06-FRGS0248-2010.

APPENDIX A

MANY MICROSCOPIC HAMILTONIANS OF MANY CONSIDERTIONS OF CLUSTERIZATIONS

The Hamiltonian operator of a system of A-nucleons nucleus in the laboratory system can be written as

$$H = \sum_{i=1}^{A} \left(\frac{\hat{p}_i^{\,2}}{2m_i} + \sum_{\substack{j=1 \\ j>i}}^{A} V_{ij} \right), \tag{A-1}$$

Where $\hat{p}_i$ and m_i are the momentum operator and the mass of the i^{th} nucleon which interacts with each j^{th} nucleon in the system by a two-body potential V_{ij}. This Hamiltonian is invariant to the permutations of position vectors $\boldsymbol{r}_i$. In some nuclear reactions and decay, the system exhibits a behavior of two groups (two clusters) in what is called the clusterization effect. If the nucleons of the system are considered in two groups with A_d nucleons and A_c nucleons, where

$$A = A_d + A_c\,, \tag{A-2}$$

where A_d are the nucleons of i = 1 to A_d and A_c are the nucleons of i = ($A - A_d$) to A. Then the Hamiltonian can be written as

$$H = \sum_{i=1}^{A_d} \left(\frac{\hat{p}_i^{\,2}}{2m_i} + \sum_{\substack{j=1 \\ j>i}}^{A} V_{ij} \right) + \sum_{i=A-A_c}^{A} \left(\frac{\hat{p}_i^{\,2}}{2m_i} + \sum_{\substack{j=1 \\ j>i}}^{A} V_{ij} \right). \tag{A-3}$$

The two outer summations are for two groups of nucleons but the potential energy terms are expanded to all nucleons. To separate this interaction it is possible to consider the kinetic energy of any nucleon as a sum of two parts, $K_i = K_{oi} + K_{1i}$; the first is due to the interaction of i^{th} nucleon with the other nucleons in its group and the second is for the nucleons in the other group. In terms of operators,

$$\hat{p}_i{}^2 = \hat{p}_{oi}{}^2 + \hat{p}_{1i}{}^2 . \quad \text{(A-4)}$$

Substituting Eq.(A-4) in and splitting the potential-energy terms of the other group from each sum in Eq.(A-3) and rearrange them, we obtain a non-invariant Hamiltonian to the permutations,

$$H = \sum_{i=1}^{A_d}\left(\frac{\hat{p}_{oi}{}^2}{2m_i} + \sum_{\substack{j=1\\ j>i}}^{A_d} V_{ij}\right) + \sum_{i=A-A_c}^{A}\left(\frac{\hat{p}_{oi}{}^2}{2m_i} + \sum_{\substack{j=A-A_c\\ j>i}}^{A} V_{ij}\right) + \sum_{i=1}^{A_d}\left[\frac{\hat{p}_{1i}{}^2}{2m_i} + \sum_{j=A-A_c}^{A}\left(\frac{\hat{p}_{1j}{}^2}{2m_j} + V_{ij}\right)\right]. \quad \text{(A-5)}$$

This equation contains three main sums. In each one there are momentum operators and potential energies which could be substituted in Schrodinger Equation (S.E.) to obtain the quantum-mechanical states and the wavefunction that describe the nucleons, so these three terms can be written in term of different Hamiltonians as

$$H = H_f(A_d) + H_f(A_c) + H_r(A_d \otimes A_c) . \quad \text{(A-6)}$$

It is important to notice that only the first two Hamiltonians are invariant to the permutation. It is possible to consider the similarity of the first two Hamiltonians as being of clusters, so they can be written together to represent the whole nucleons of the system. Then, the Hamiltonian responsible for the formation can be set as

$$H_f = H_f(A_d) + H_f(A_c) . \quad \text{(A-7)}$$

The Hamiltonian of Eq.(A-6) can be written as a sum of two $H = H_f + H_r$; one for the clusters formation and the other for the interaction between the two clusters. Writing them in the Time-Independent Schrodinger Equation (TISE), we obtain

$$H\Psi = (H_f + H_r)\Psi . \quad \text{(A-8)}$$

This equation can be separated in accordance to the Hamiltonians of Eq.(A-6) and the total wavefunction of the system can be written as

$$\Psi = \Phi_d(\xi)\Phi_c(\eta)\Phi_{dc}(\rho) . \quad \text{(A-9)}$$

The three wavefunctions are supposed to be defined on their space coordinates ξ, η and ρ, and found from the solution of TISEs;

$$H_{fd}\Phi_d(\xi) = E_{fd}\Phi_d(\xi), H_{fc}\Phi_c(\eta) = E_{fc}\Phi_c(\eta), H_r\Phi_{cd}(\rho) = E_r\Phi_{dc}(\rho). \quad \text{(A-10)}$$

These wavefunctions are eigenfunctions to the operators; H_{fd}, H_{fc} and H_r with eigenvalues E_d, E_c, and E_{dc} respectively. The first two equations in

Eq.(A-10) are related to the cluster-formation Hamiltonian $H_f = H_{fd} + H_{fc}$, so, Eq.(A-8) can be

$$H\Psi = (E_d + E_c + E_{dc})\Psi = (E_f + E_{dc})\Psi = E\Psi . \quad \text{(A-11)}$$

The total energy E is the eigenvalue of the Hamiltonian of Eq.(A-6). The reformulation of the Hamiltonian Eq.(A-1) into Eq.(A-6) enables us to write the Hamiltonian into different forms depending on the consideration of the different clusterizations.

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