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ON THE PHYSICAL INTERPRETATION OF THE ELECTROMAGNETIC COUPLING CONSTANT α

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Abstract. The analysis of the electromagnetic and nuclear interactions between particles with zero kinetic energies or particles in bound states supports the idea that the interaction strength is proportional to the energy (mass) of the interaction field: electrostatic field and mesonic field respectively. Starting from this analysis a unitary physical interpretation of the electromagnetic coupling constant α and the strong coupling constant α_s is proposed. It is shown on a simple model that α is the ratio between the electrostatic energy in the electric field produced by a unit electric charge distributed on the surface of a sphere and the minimum energy given by the uncertainty relation for one degree of freedom, of current mass $\equiv 0$, confined inside the same sphere. Moreover, the ratio between the densities of these two energies is α in any point. The value obtained for the ratio between the electromagnetic interaction strength and the nuclear interaction strength is lower than 1/100, as generally accepted.

Key words: coupling constant α , electromagnetic mass, dynamical mass, ratio between electromagnetic and nuclear interaction strengths.

1. INTRODUCTION

The electromagnetic coupling constant, or fine structure constant:

$$\alpha = \frac{e^2}{\hbar c 4\pi\epsilon_0} = \frac{1}{137} \quad (1)$$

is a fundamental physical constant, which characterizes the strength of the electromagnetic interaction. In electrostatic cgs units the fine-structure constant becomes $\alpha = e^2/\hbar c$. There are several well-known physical interpretations of this fine-structure constant: the ratio of the velocity of the electron in the first Bohr orbit of hydrogen atom to the speed of light in vacuum; the square of the ratio of the electric charge to the Planck charge $(e/q_P)^2$; the ratio of two energies: the

energy needed to bring two electrons from infinity to a distance r against their electrostatic repulsion, and the energy of a single photon of wavelength equal to $2\pi r$.

In the present work we try to give a new physical interpretation to this coupling constant starting from an analysis of the interaction between particles with zero kinetic energies or particles in bound states. A unitary interpretation of the electromagnetic coupling constant and strong coupling constant is proposed.

The mass defects of atoms and nuclei, which are equal to their binding energies, are proportional to the strength of the interaction. Since there is a direct correlation between these mass defects and the decrease of the mass (energy) of the interaction fields of the particles bound into atoms or nuclei, electromagnetic field for leptons and mesonic field for nucleons, the interaction strength is proportional to the mass (energy) of the interaction field.

Let's analyze the Coulomb interaction. Two electric charges of opposite signs, for instance electron and positron, which are each other at a distance r , are characterized by an interaction energy equal to $-e^2/r$. The total electrostatic energy in the electric fields produced by the two charges, and therefore the sum of the electromagnetic masses of electron and positron, is lower with a value equal to e^2/r as compared to the case the two leptons are at an infinite distance. The electrostatic mass of each lepton decreased with $e^2/2r$. In the case of e^+e^- bound state the mass defect (binding energy) is of the order of 7 eV. For maximum "approach", this means the e^+e^- annihilation process, the "mass defect" gets maximum: $2m_e c^2$.

In the case of nuclear interaction, the binding energy (mass defect) per nucleon is about 7 MeV. The attraction between two nucleons is given mainly by two-pion exchange [1]. Taking into account that the current masses of quarks (antiquarks) u (\bar{u}) and d (\bar{d}) are very small the pion mass is mainly a "dynamical" one, due to confinement of these quarks inside a region of radius $r \sim 1$ fm. At the formation of a nuclear bound state, this dynamical mass decreases. Indeed a degree of freedom of current mass $\cong 0$ (u or d quark) confined inside the periphery of a nucleon of radius r_N associates a minimum energy (dynamical mass) $E \cong pc \geq \hbar c/2r_N$, given by the Heisenberg uncertainty relation. When the two nucleons approach each other to form a bound state, for instance the deuteron, they put in common their pionic peripheries which means that some degrees of freedom will slightly de-confine from a region of radius r_N to a region of radius $\sim r_N + \Delta r$, where Δr is the distance between the two bound nucleons. The new dynamical mass $\hbar c/(2r_N + \Delta r)$ is lower than the initial one (that in the free nucleon). To form a bound state the total decrease of dynamical masses must be larger than the kinetic energies acquired by nucleons due to localization.

This mechanism of nuclear binding has similarities with the molecular binding of diatomic molecules. For instance, in the hydrogen molecular ion, since there are two protons, there is more space where the electron can have a low potential energy than in the case of hydrogen atom; the electron spreads out

lowering its kinetic energy, without increasing its potential energy. This energy decrease contributes to the binding energy of hydrogen molecular ion [2].

2. THE ANALYSIS OF A SIMPLE MODEL

Let's compare the energy of the electromagnetic field to the dynamical mass on a simple model: the energy in the electric field E generated by a unit charge uniformly distributed on a sphere of radius r and the energy (dynamical mass) of one degree of freedom, with current mass $\cong 0$, confined inside the same sphere of radius r (Fig. 1).

The electrostatic energy density is given by:

$$u_{el} = \frac{E^2}{8\pi} = \frac{e^2}{8\pi \cdot r^4}. \quad (2)$$

To get the total electrostatic energy one must integrate over all space:

$$U_{el} = \int_r^\infty u_{el} 4\pi \cdot r^2 dr = \frac{e^2}{2r}. \quad (3)$$

On the other hand, one degree of freedom, with current mass $\cong 0$, confined inside the same sphere of radius r associates a minimum energy (dynamical mass) $\hbar c/2r$.

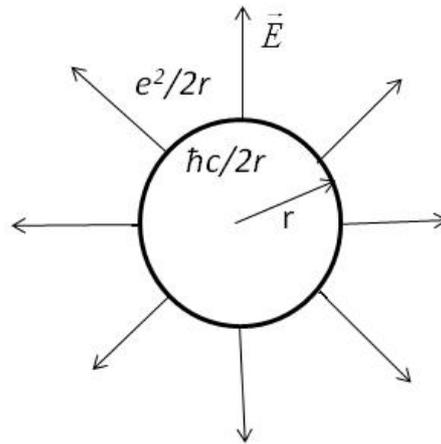


Fig. 1 – The energy in the electric field E generated by a unit charge uniformly distributed on the sphere of radius r and the energy (dynamical mass) of one degree of freedom, with current mass $\cong 0$, confined inside the same sphere of radius r .

The ratio between these two energies is:

$$\frac{\frac{e^2}{2r}}{\frac{\hbar c}{2r}} = \frac{e^2}{\hbar c} \equiv \alpha, \quad (4)$$

that is just the coupling constant α .

Since both energies $e^2/2r$ and $\hbar c/2r$ have a $1/r$ dependence, the corresponding energy densities $e^2/8\pi r^4$ and $\hbar c/8\pi r^4$ have the same dependence on r . This means that the ratio between these two energy densities is α in any point r .

Let's compare both energies, the electrostatic energy $e^2/2r$ and the dynamical mass, to the same "standard" energy given by the uncertainty relation: the minimum energy $\hbar c/2r$ of one degree of freedom, of current mass $\cong 0$, confined inside a sphere of radius r .

Obviously, for electromagnetic interaction one obtains the relation (4) for the coupling constant α .

For nuclear interaction, one obtains the coupling constant:

$$\alpha_s^{(1)} = \frac{\frac{\hbar c}{2r}}{\frac{\hbar c}{2r}} = 1, \quad (5a)$$

in the case of a hypothetical hadron composed of one degree of freedom. For two degrees of freedom, two quarks confined in the same volume (the pion), the coupling constant is:

$$\alpha_s^{(2)} = \frac{2 \frac{\hbar c}{2r}}{\frac{\hbar c}{2r}} = 2. \quad (5b)$$

The ratio between the electromagnetic and nuclear interaction strengths is $\alpha/\alpha_s^{(1)}$, this means lower than 1/100 as generally accepted [3].

Let's take as confinement radius the Compton wavelength of the charged pion $r = \lambda_\pi = \hbar/m_\pi c$. It results:

$$\frac{\hbar c}{2r} = \frac{\hbar c}{2\lambda_\pi} = \frac{m_\pi c^2}{2}. \quad (6)$$

As the pion has two degrees of freedom, $m_\pi c^2/2$ is the dynamical mass due to one quark (one degree of freedom).

It is interesting to note that the electrostatic energy produced by a unit electric charge on a sphere of radius $r = \lambda_\pi \cong 1.4$ fm is equal to $e^2/2\lambda_\pi \cong m_e c^2$, i.e. the electron mass.

From relation (4) one obtains:

$$\alpha = \frac{\frac{e^2}{2\hat{\lambda}_\pi}}{\frac{\hbar c}{2\hat{\lambda}_\pi}} \cong \frac{m_e c^2}{\frac{m_\pi c^2}{2}} = \frac{1}{136.56}. \quad (7)$$

Hence α is, in a good approximation, the ratio between the mass of the lightest charged lepton and half the mass of the lightest meson, the charged pion. This means that relation (7) suggests that the greatest part of the electron mass would be of electrostatic origin. How much of the electron mass is electromagnetic represents an old subject of debate in physics [4, 5]. In any case, it is meaningless to separate a charge from its Coulomb field. Neither can be observed without the other; a charge at rest is always surrounded by a Coulomb field and conversely every Coulomb field has a source [5]. Coulomb field means electrostatic energy and consequently electrostatic mass for electron.

3. DISCUSSION AND CONCLUSIONS

The analysis of the electromagnetic and nuclear interactions between particles in bound states supports the idea that the interaction strength is proportional to the energy (mass) of the interaction field: electrostatic field and mesonic field respectively.

The electromagnetic coupling constant α is the ratio between the electrostatic energy due to a unit charge uniformly distributed on a sphere of radius r and the “standard” energy given by the uncertainty relation: the minimum energy $\hbar c/2r$ of one degree of freedom, of current mass $\cong 0$, confined inside the same sphere of radius r . Since the corresponding energy densities $e^2/8\pi r^4$ and $\hbar c/8\pi r^4$ have the same dependence on r , their ratio is also α in any point. This means that α is the coupling constant in any vertex corresponding to an electromagnetic interaction.

We have shown in the introduction that the dynamical mass, generated by confinement of some degrees of freedom inside the nucleon, is at the origin of the nuclear interaction. Indeed, when two nucleons approach each other to form a bound state they put in common pionic peripheries, which means a slight de-confinement of some degrees of freedom. This means a decrease of their dynamical masses, which contributes directly to the binding energy of the two nucleons. This mechanism reflects a clear connection between confinement and nuclear interaction (residual strong interaction) *via* dynamical mass. The value of about 1 fm of the confinement radius determines the values of the dynamical masses and from here the strength of the nuclear interaction. The strong coupling constant $\alpha_s^{(i)}$ is the ratio between the dynamical mass and the “standard” energy given by the

uncertainty relation inside the same sphere of radius r , and has a value ≥ 1 . The leptons have no dynamical mass therefore they do not participate to strong interaction.

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