ON TRANSITION DENSITIES IN PHENOMENOLOGICAL NUCLEAR STRUCTURE MODELS

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Abstract. The Complete Nuclear Spectroscopy is using the Nuclear Reactions/Scattering in order to extract Nuclear Structure information. The channel coupling matrix elements of DWBA or of Channel Coupling methods are parametrized in terms of Spectroscopic Factors and of Transition Densities (related to spatial dependence of interaction too). The transition densities became, in Complete Nuclear Spectroscopy, an essential instrument for extracting spectroscopic information via hadronic (or electron) excitations of the atomic nucleus. In this work one discusses comparatively the transition densities in phenomenological geometrical and algebraic collective nuclear structure models.

Key words: Phenomenological Nuclear Structure Models, transition densities.

1. INTRODUCTION

The atomic nucleus is usually studied by means of γ-ray Spectroscopy, *e.g.* [1]; alternative methods for nuclear structure studies are based on electron scattering, *e.g.* [2] (or [1]) and on nuclear scattering/reactions, *e.g.* [3]. The nuclear structure investigations by means of hadronic excitation are very important in the Complete Nuclear Spectroscopy, *e.g.* [4]; the nuclear reaction is used as an instrument for extracting nuclear structure information. The accurate description of the nuclear reaction dynamics is consequently a vital step in these studies. The Nuclear Reactions Models as DWBA or Channel Coupling method are used for describing Direct Reactions, [3]. In both methods the Transition Matrix elements are parametrized in terms of the Spectroscopic Amplitude, Form Factors and (quantum) kinematical factors. The spatial dependence of the interaction (Form Factor) between hadronic probe and atomic nucleus is described in terms of Transition Densities; this way the Transition

Densities became component of the potential inducing transitions between different reaction channels. The Transition densities, as component of transition potential, consists both of spectroscopic factor (nuclear structure information) and of form factor (radial dependence of the hadron-nucleus interaction).

Relations between electromagnetic multipole moments and nuclear (potential) multipole moments (obtained by integrating with respect to radius of the transition densities) are known as Satchler Theorem, [3]. There exist also relations between electromagnetic multipole moments and the Coulomb multipole moments, (involved, for example, in electron scattering); they are subject of Siegert Theorem, [2]. The two theorems are fundamental in comparison of the nuclear spectroscopic data, obtained by means of different probes: electromagnetic transitions, nuclear reactions or electron scattering.

The transition densities are specific for nuclear structure studies done either via nuclear reactions (hadronic excitation) or by electron scattering on atomic nuclei. In this work the problem of transition densities is approached, in a comparative way, both for geometric and algebraic collective models.

2. NUCLEAR, ELECTROMAGNETIC AND COULOMB MULTIPOLE MOMENTS

The potential describing interaction between the hadron-projectile and target-nucleus depends both on geometrical coordinates (r, θ, ϕ) as well as on internal nuclear coordinates, [5]. In geometrical models the internal coordinates are just parameters of the nucleus surface. For a vibrational nucleus, the surface radius is described by,

$$R(\theta,\phi) = R_0[1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta,\phi)],$$

where $\alpha_{\lambda\mu}$ are dynamical parameters describing surface vibrations around spherical shape R_0. The most important vibrations are the quadrupole ($\lambda = 2$) and octupole ($\lambda = 3$) ones. For a rotational nucleus, permanently deformed, the radius dependence on nucleus spatial orientation is

$$R(\theta,\phi) = R_0[1 + \sum_{\lambda} \beta_{\lambda} Y_{\lambda 0}(\theta')],$$

where the angle θ' refers to the body-fixed coordinate system, related to laboratory system angles (θ, ϕ) by the rotation matrix elements D for Euler angles θ_i, $Y_{\lambda 0}(\theta').= \sum D_{\mu 0}^{\lambda}(\theta_i) Y_{\lambda\mu}(\theta,\phi)$. In both cases the interacting potential is considered a deformed Woods-Saxon form and its derivative.

The interacting potential is divided into a spherical shape part and a non-

spherical part related to components as $\alpha_{\lambda\mu}Y_{\lambda\mu}$ or $\beta_\lambda Y_{\lambda 0}$, [5],

$$V(r,\theta,\phi) = V_{diag}(r) + V_{coupl}(r,\theta,\phi).$$

The diagonal component $V_{diag}(r)$ is responsible for elastic scattering while the non-diagonal component $V_{coupl}(r,\theta,\phi)$ induces transitions between different reaction channels. The coupling part of the potential is expanded into multipoles

$$V_{coupl}(r,\theta,\phi) = \sum_{\lambda\mu} V_{\lambda\mu}(r)Y_{\lambda\mu}(\theta,\phi)$$

$$V_{\lambda\mu}(r) = v_\lambda(r)Q_{\lambda\mu}$$

In case of Vibrational Model the multipole expansion is obtained by developing the Woods-Saxon form into powers of the quantity $t(\theta\phi) = \sum_{\lambda\mu}\alpha_{\lambda\mu}Y_{\lambda\mu}$, which defines the non-spherical part of the nuclear radius

$$R(\theta,\phi) = R_0[1 + \sum_{\lambda\mu}\alpha_{\lambda\mu}Y_{\lambda\mu}] = R_o[1 + t(\theta,\phi)].$$

The multipole components can be further represented in terms of first-order, second-order transitions

$$V_{\lambda\mu}(r) = \sum_t v_\lambda^t(r)Q_{\lambda\mu}^t$$

with $Q_{\lambda\mu}^1 = \alpha_{\lambda\mu}$ and $Q_{\lambda\mu}^2$ proportional to product $(\alpha_{\lambda_1} \times \alpha_{\lambda_1})_{\lambda\mu}$.

$$Q_{\lambda\mu}^{(1)} = \alpha_{\lambda\mu}$$
$$Q_{\lambda\mu}^{(2)} = \sum_{\lambda_1\lambda_2} C(\lambda_1,\lambda_2,\lambda) <\lambda_1\lambda_2 00|\lambda 0> (\alpha_{\lambda_1} \times \alpha_{\lambda_1})_{\lambda\mu}$$

The nuclear structure information is contained in nuclear reduced matrix elements

$$< 0||Q_{\lambda\mu}^{(1)}||I > = \beta_\lambda \delta_{\lambda I}$$
$$< 0||Q_{\lambda\mu}^{(2)}||I > \sim \beta_{(2nd)}$$

with $\beta_{(2nd)}$ proportional to β^2.

For rotational nuclei, strongly deformed, β_λ large, the power expansion is slow convergent and one uses the angular integration for obtaining the potential multipoles $V_{\lambda\mu}(r)$; they are not more represented as first-order and second-order transitions. The method was extended to Triaxial Rotor Model nuclei too, [6], [7], [8].

3. TRANSITION DENSITIES IN GEOMETRIC COLLECTIVE NUCLEAR MODELS

The multipole expansion of the interacting potential

$$V_{coupl}(r,\Omega) = \sum_{\lambda\mu} V_{\lambda\mu}(r)Y_{\lambda\mu}(\Omega) = \sum_{\lambda\mu} v_\lambda(r)Q_{\lambda\mu}Y_{\lambda\mu}(\Omega)$$

is used to define the Nuclear Multipole components, [9], [3], by integrals

$$M_{\lambda\mu} = \int V(r,\Omega)Y_{\lambda\mu}(\Omega)r^{\lambda+2}\,\mathrm{d}r\,\mathrm{d}\Omega = q_\lambda Q_{\lambda\mu}$$

with radial transition potential moments defined by $q_\lambda = \int v_\lambda r^{\lambda+2}\,\mathrm{d}r$.

The Transition Densities are defined as reduced matrix elements of the Transition Operator, taken between two nuclear states labeled by their spins I and I'

$$< I||V_\lambda||I' > = v_\lambda(r) < I||Q_\lambda||I' > .$$

The multipoles are obtained by radial integration

$$< I||M_\lambda||I' > = q_\lambda < I||Q_\lambda||I' > .$$

The normalized multipoles (to the volume integral of interaction) become

$$< I||M_\lambda||I' > / \int Vr^2\,\mathrm{d}r\,\mathrm{d}\Omega = q_\lambda^n < I||Q_\lambda||I' >$$

with normalized radial transition operator moments:

$$q_\lambda^n = \int v_\lambda(r)r^{\lambda+2}\,\mathrm{d}r / \int Vr^2\,\mathrm{d}r\,\mathrm{d}\Omega.$$

The multipole moments for emission or absorption of γ-rays of multipolarity $(E\lambda\mu)$, are defined by

$$M(E\lambda\mu) = \int \rho_e(r,\Omega)Y_{\lambda\mu}(\Omega)r^{\lambda+2}\,\mathrm{d}r\,\mathrm{d}\Omega.$$

By assuming that the charge density $\rho_e(r,\Omega)$ follows the mass density $\rho(r,\Omega)$ dependence, $\rho_e(r,\Omega) = Ze/A\,\rho(r,\Omega)$ one obtains the Satchler theorem, [3],

$$M(E\lambda\mu)/Ze = M_{\lambda\mu}/ \int Vr^2\,\mathrm{d}r\,\mathrm{d}\Omega.$$

The Satchler theorem assumes that the Optical Model Potential, used to generate the potential and mass moments, is obtained by a folding procedure with an interaction which is central, scalar and independent on local density.

The Coulomb moments are related to Coulombian interaction of a particle penetrating atomic nucleus

$$M(C\lambda\mu) = \int \rho(r,\Omega)f_\lambda(r)Y_{\lambda\mu}(\Omega)\,\mathrm{d}r\,\mathrm{d}\Omega,$$

where $f_\lambda(r)$ defines the radial motion of the particle inside nucleus. The Siegert theorem, [2], is obtained when the radial wave function can be replaced by the Bessel function. It states that Coulomb and electric multipoles are proportional provided the multipolarity λ is equal or larger than 2.

$$M(C\lambda\mu) = M(E\lambda\mu)$$

The transition densities in the Geometric Models follow naturally from (generalized) Woods-Saxon form of the interacting potential. The radial dependence of the transition densities is either Woods-Saxon first-order derivative, $v_\lambda^{(1)}(r)$, (if first order transitions), or Woods-Saxon second-order derivative, $v_\lambda^{(2)}(r)$, (if they refer to second order transitions):

$$v_\lambda^{(1)}(r) = -(R_0/a)e(1+e)^{-2},$$
$$v_\lambda^{(2)}(r) = 1/2(R_0/a)^2 e(1-e)(1+e)^{-3},$$

where Woods-Saxon form, $(1+e)^{-1}$, is defined by $e = \exp[(r - R_0/a)]$, with R_0 - nuclear radius and a - nucleus diffuseness.

4. TRANSITION DENSITIES IN ALGEBRAIC COLLECTIVE NUCLEAR MODELS

The Algebraic Nuclear Models assume a Hamiltonian constructed by means of Group Theoretic Techniques. The Interacting Boson Model-1 [10] Hamiltonian has $U(6)$ group structure and it is constructed in terms of generators of $U(6)$ algebra. The Interacting Boson Model-1 assumes a six-dimensional space spanned by the $s - d$ boson system. The two kinds of bosons, s-boson $(L = 0)$ and d-boson $(L = 2)$ have one, respectively, five magnetic substates. The total number of s- and d- bosons is conserved. The bosons have one-body energies: ϵ_s and ϵ_d. The interaction between bosons are two-body interactions. The Hamiltonian is constructed in terms of one-body and two-body operators, generators of the $U(6)$ algebra, which at their turn are products of boson creation and annihilation operators: s^+ and s and d_μ^+ and d_μ, μ = +2, +1, 0, -1, -2; it is useful to define annihilation operators that behave as tensor operator of rank zero, $\tilde{s}$, and tensor operator of rank two, $\tilde{d}$, by: $\tilde{s} = s$ and $\tilde{d}_\mu = (-1)^\mu d_{-\mu}$. The Hamiltonian must be rotational invariant, therefore the rotational $O(3)$ group is present in the decomposition group chains. This condition results in three group chains, namely $U(5)$, $O(6)$ and $U(3)$. They correspond to Vibrational, Jean-Willets and Rotational geometrical models. The operators for electromagnetic

transitions are one-body operators, [10]:

$$T(E0) = \eta_0 + \gamma_0 \cdot N + \beta_0 \cdot n_d,$$
$$T_\mu(E2) = \alpha_2 [d^+ \times \tilde{s} + s^+ \times \tilde{d}]_\mu^{(2)} + \beta_2 [d^+ \times \tilde{d}]_\mu^{(2)},$$
$$T_\mu(E4) = \beta_4 [d^+ \times \tilde{d}]_\mu^{(4)},$$
$$n_s = [s^+ \times \tilde{s}]^{(0)},$$
$$n_d = [d^+ \times \tilde{d}]^{(0)},$$
$$N = n_s + n_d.$$

The parameters α, β and γ are coupling constants of bosons to external field and are empirical ones. The matrix elements of transition operators, evaluated between two nuclear states i and $1(j)$, define the electromagnetic transition rates. The reduced matrix elements are:

$$A_{1i}^{(2)} = <2_i^+ ||[d^+ \times \tilde{s} + s^+ \times \tilde{d}]^{(2)}||0_i^+ >,$$
$$B_{1i}^{(2)} = <2_i^{(+)} ||[d^+ \times \tilde{d}]^{(2)}||0_1^+ >,$$
$$B_{1i}^{(4)} = <4_i^{(+)} ||[d^+ \times \tilde{d}]^{(4)}||0_1^+ >,$$
$$B_{ji}^{(0)} = <0_i^{(+)} ||[d^+ \times \tilde{d}]^{(0)}||0_j^+ >.$$

The reduced matrix elements together with α, β, γ coupling parameters and $e_{SD}^{(2)}$, $e_{DD}^{(2)}$, $e_{DD}^{(4)}$ effective charges, define the transition matrix elements

$$M(E2) = e_{SD}^{(2)} \alpha_2 A_{1i}^{(2)} + e_{DD}^{(2)} \beta_2 B_{1i}^{(2)},$$
$$M(E4) = e_{DD}^{(4)} \beta_4 B_{1i}^{(4)}.$$

The nuclear states could be also excited by electron or hadron scattering; now the coupling parameters to the external field α, β, γ become radial dependent. In addition to strength of interaction, the coupling parameters describe the spatial dependence of interaction too. They are very similar to the electromagnetic transition operators:

$$T(l = 0) = \eta_0(r) + \gamma_0(r) \cdot N + \beta_0(r) \cdot n_d,$$
$$T(l = 2) = \alpha_2(r)[d^+ \times \tilde{s} + s^+ \times \tilde{d}]^{(2)} + \beta_2(r)[d^+ \times \tilde{d}]^{(2)},$$
$$T(l = 4) = \beta_4(r)[d^+ \times \tilde{d}]^{(4)}.$$

The transition densities are reduced matrix elements of transition operators

evaluated between the nuclear states:

$$\rho_{0^+2^+}^{(2)} = e_{SD}^{(2)}\alpha_2(r)A_{1i}^{(2)} + e_{DD}^{(2)}\beta_2(r)B_{1i}^{(2)},$$
$$\rho_{0^+4^+}^{(4)} = e_{DD}^{(4)}\beta_4(r)B_{1i}^{(4)},$$
$$\rho_{0^+0^+}^{(0)} = e^{(0)}\beta_0(r)B_{ji}^{(0)}.$$

The interaction spatial dependent functions $\alpha_2(r)$, $\beta_2(r)$, $\beta_4(r)$ and $\beta_0(r)$ and $\rho^{(o)}$ transition density are normalized to result in

$$\alpha_2 = \int r^4\alpha_2(r)\,\mathrm{d}r,$$

$$\beta_2 = \int r^4\beta_2(r)\,\mathrm{d}r,$$

$$\beta_4 = \int r^6\beta_4(r)\,\mathrm{d}r,$$

$$\beta_0 = \int r^4\beta_0(r)\,\mathrm{d}r,$$

$$Z = \int 4\pi r^2\rho_{0_10_1}^{(0)}\,\mathrm{d}r.$$

It follows their connection to electromagnetic transition rates; for example

$$M(E2) = e_{SD}^{(2)}\int r^4\alpha_2(r)\,\mathrm{d}r A_{1i}^{(2)} + e_{DD}^{(2)}\int r^4\beta_2(r)\,\mathrm{d}r B_{1i}^{(2)}.$$

5. TRANSITION DENSITIES IN VIBRATIONAL MODEL

Let us compare the transition densities defined in Geometric and Algebraic Collective Nuclear Models. For example the $M(E2)$, $\lambda = 2$, transition matrix element in Vibrational Model is

$$M(E2)/Z = \beta_2 \int r^{\lambda+2}V_\lambda^{tr}(r)\,\mathrm{d}r / \int r^2 V_{opt}(r)\,\mathrm{d}r$$

with V_λ^{tr} - transition potentials; they are $v_\lambda^{(i)}(r)$ if refer to first-order $i = 1$ transitions or second-order $i = 2$ transitions. One can observe that

$$e_{SD}^{(2)}\int r^4\alpha_2(r)\,\mathrm{d}r / \int 4\pi r^2\rho_{(0^+0^+)}^{(0)}(r)\,\mathrm{d}r \sim \int v_\lambda^{(1)}r^4\,\mathrm{d}r / \int 4\pi r^2 V_{opt}(r)\,\mathrm{d}r,$$

$$e_{DD}^{(2)}\int r^4\beta_2(r)\,\mathrm{d}r / \int 4\pi r^2\rho_{(0^+0^+)}^{(0)}(r)\,\mathrm{d}r \sim \int v_\lambda^{(2)}r^4\,\mathrm{d}r / \int 4\pi r^2 V_{opt}(r)\,\mathrm{d}r.$$

One concludes, from this analogy, that the algebraic transition densities are – up to normalization constant – just first and second order derivative of geometric form-

factors. Observe the similarity of algebraic transition densities

$$\rho_{0^+2^+}^{(2)} = e_{SD}^{(2)}\alpha_2(r)A_{1i}^{(2)} + e_{DD}^{(2)}\beta_2(r)B_{1i}^{(2)},$$
$$\rho_{0^+4^+}^{(4)} = e_{DD}^{(4)}\beta_4(r)B_{1i}^{(4)}$$

to the transition potentials of geometrical models

$$V_{0^+2^+}^{(2)} = \beta_2 V'(r) + \beta_{2nd}V''(r),$$
$$V_{0^+4^+}^{(4)} = \beta_{(4)}V'(r).$$

This analogy between Form-Factors of Geometrical Vibrational Model and the Form-Factors of Algebraic Transition Densities can have a more profound physical meaning. The nuclear structure information occurs only in reduced matrix elements A and B. The Algebraic Form-Factors as $\alpha_2(r)$ and $\beta_2(r)$ do not depend on involved nuclear states; they do not depend on number N or types (n_s or n_d) of involved bosons. The Form-Factors are unmodified even in large limit of bosons; in the $U(5)$ variant this limit is just Vibrational Model (VM). The non-conserving d-boson term $[d^+ \times s + s^+ \times d]^2$ is corresponding of the $Q_2^{(1)}$ VM multipole operator, which is also a non-conserving quadrupole boson operator. The d-boson conserving term $[d^+ \times d]$ has a corresponding in quadrupole boson conserving term from VM multipole operator $Q_2^{(2)}$. This term in $Q^{(2)}$ is responsible for transitions within same multiplet of Vibrational Model; its complementary part in $Q_2^{(2)}$ produces transitions between VM multiplets differing by two quadrupole bosons. One can conclude that $\alpha_2(r)$ should be a Form-Factor similar to that multiplying $Q_2^{(1)}$ VM multipole operator, *i.e.* first Woods-Saxon derivative, while $\beta_2(r)$ should be similar to the Form-Factor multiplying $Q_2^{(2)}$ that is second Woods-Saxon derivative.

The above justification for Algebraic Transition Densities is valid only in $U(5)$ Vibrational limit. There is no physical reason to extend it in this form to $O(6)$ or $U(3)$ limits. One can try however to construct similar procedures to Rotational $U(3)$ limit but then it is no more possible a clear-cut separation of different form-factors, [11], but rather a complicated combination of form-factors and of spectroscopic amplitudes in multistep transitions.

6. TRANSITION DENSITIES IN ROTATIONAL MODEL

In the work [11] one follows the power expansion of the interacting potential with a deformed rotational nucleus, procedure developed in [12]. The next step was to use a relationship between IBM and Bohr-Mottelson collective model by means of the intrinsic state defined in [13]. This intrinsic state which allows to establish a

relation between IBM and the collective rotational model is

$$|N, \overline{\beta}> = \frac{1}{\sqrt{N!(1+\overline{\beta}^2)^N}} [s^+ + \overline{\beta} d_0^+]^N |0>,$$

where $|0>$ is the state with zero bosons, and $\overline{\beta}$ is describing the deformation in the large N limit, but it is not identical with the deformation parameter of the collective model [13].

The quadrupolar transition operator is usually written as

$$T_\mu(E2) = \alpha_2(r)[d^+ \times s + s^+ \times \tilde{d}]_\mu^{(2)} + \beta_2(r)[d^+ \times \tilde{d}]_\mu^{(2)},$$

where the form factors α_2 and β_2 may depend on rotational scalars: the number of scalar bosons, $s^+ s$, and the number of quadrupolar bosons, $d^+ \tilde{d}$. The matrix elements of these operators between intrinsic states are [13]:

$$< N, \overline{\beta}|s^+ s|N, \overline{\beta}> = \frac{N}{1+\overline{\beta}^2},$$

$$< N, \overline{\beta}|d^+ \tilde{d}|N, \overline{\beta}> = \overline{\beta}^2 \frac{N}{1+\overline{\beta}^2}$$

and their form suggest that the form factors should depend on the ratio $d^+ \tilde{d}/s^+ s$ as it is proportional with the deformation parameter $\overline{\beta}^2$. Evaluating the transition operator between the intrinsic states in the large N limit, and comparing with the transition operator in the geometric model, one obtains form factors involving arbitrarily high order derivatives of the Woods-Saxon function [11]:

$$\alpha_2(r, \overline{\beta}^2) = \frac{1}{N}(1+\overline{\beta}^2) \sum_{k=0}^{\infty} \frac{(-aNR_0)^{2k+1}(\overline{\beta}^2)^k}{(2k+1)!} \gamma_2^{(2k+1)} \frac{\partial^{2k+1}}{\partial r^{2k+1}} U(r - R_0),$$

$$\beta_2(r, \overline{\beta}^2) = \frac{1}{N}(1+\overline{\beta}^2) \sum_{k=0}^{\infty} \frac{(-aNR_0)^{2k+2}(\overline{\beta}^2)^k}{(2k+2)!} \gamma_2^{(2k+2)} \frac{\partial^{2k+2}}{\partial r^{2k+2}} U(r - R_0),$$

where $\gamma_L^{(n)}$ are given in terms of the Clebsch-Gordan coefficients using the recurrence relation

$$\gamma_L^{(n)} = \sum_\Lambda \left[\frac{(2\Lambda + 1)(2L + 1)}{4\pi} \right]^{1/2} \begin{pmatrix} \Lambda & 2 & L \\ 0 & 0 & 0 \end{pmatrix} \gamma_\Lambda^{(n-1)}$$

and $\gamma_L^{(1)} = \delta_{L2}$. Therefore, the lowest order approximation in the deformation pa-

rameter $\overline{\beta}^2$ for the quadrupolar transition operator $T(E2)$ is

$$T_\mu(E2) = -aR_0 \frac{\partial}{\partial r} U(r - R_0)[d^+ s + s^+ \tilde{d}]_\mu^{(2)} +$$

$$Na^2 \frac{3 \begin{pmatrix} 2 & 2 & 2 \\ 0 & 0 & 0 \end{pmatrix}}{2\sqrt{4\pi}} R_0^2 \frac{\partial^2}{\partial r^2} U(r - R_0)[d^+ \tilde{d}]_\mu^{(2)}.$$

The non-conserving quadrupole boson-term is not dependent on total boson number while the quadrupole boson conserving term is proportional to total boson number [11]. The case of high order multipole operators can be discussed in a similar way.

7. CONCLUSIONS

The problem of Transition Densities in Algebraic Nuclear Models is far to be a simple one. On other hand it is vital in extracting reliable nuclear structure information, in Complete Nuclear Spectroscopy, by means of nuclear reactions or of electron scattering. The postulated forms for Transition Densities [10] could be justified only for case of large number of bosons and in $U(5)$ Vibrational limit. It cannot be extended in a simple way to other limits as, for example, $U(3)$ Rotational limit; a clear-cut separation of different form-factors, is not more possible, resulting in a complicated combination of form-factors and of spectroscopic amplitudes originating in multistep transitions. It seems also that number of bosons could have also effects on the exact forms of Transition Densities. The problem of Transition Densities requires further investigations because it could alter the accuracy of extracted spectroscopic information by means of hadronic (or electronic) excitation of nuclear states.

In the present work we propose a form for the quadrupolar transition operator in the IBM model. The lowest order approximation involves a part which does not conserve the number of quadrupolar bosons having a first order derivative of the Woods-Saxon function as a form factor. This part does not depend on the boson number. The other part which conserves the number of quadrupolar bosons has a second order derivative of the Woods-Saxon function as the relevant form factor and it is proportional with the total number of bosons.

The main advantage of this treatment is that one can obtain explicit forms for the radial dependence of the coupling constants of bosons with the external field. In the case of the coupling with the electromagnetic field, the radial integrals $\alpha_2 = \int r^4 \alpha_2(r)\, dr$ and $\beta_2 = \int r^4 \beta_2(r)\, dr$ provide the electromagnetic coupling constants. They result into the electromagnetic transition matrix elements.

REFERENCES

1. J.M. Eisenberg, W. Greiner, "*Nuclear Theory: Excitation Mechanisms of the Nucleus*" (North-Holland, Amsterdam, 1970).
2. H. Uberall, "*Electron Scattering from Complex Nuclei*" (Academic Press, New York, 1971).
3. G.R. Satchler, "*Direct Nuclear Reactions*" (Clarendon Press, Oxford, 1983).
4. R. Hertenberger *et al*, Nucl. Phys. **A154**, 414 (1994).
5. T. Tamura, Rev. Mod. Phys. **37**, 679 (1965).
6. T. Tamura, Nucl. Phys. **73**, 241 (1965).
7. H. Rebel *et al*, Nucl. Phys. **A218**, 13 (1974).
8. R. Neu *et al*, Phys. Rev. **C39**, 2145 (1989).
9. A. Bohr, B. Mottelson, "*Nuclear Structure*", vol. II (Benjamin, New York, 1975).
10. F. Iachello, A. Arima, "*Interacting Boson Model*" (Cambridge Univ. Press, Cambridge, 1987).
11. R.A. Ionescu, C. Hategan, Rom. Rep. Phys. **50**, 437 (1998).
12. N.K. Glendening, "*Direct Nuclear Reactions*" (Academic Press, New York, 1983).
13. J.N. Ginocchio, M.W. Kirson, Nucl. Phys. **A350**, 31 (1980).