INTERFEROMETRIC VIBRATION DISPLACEMENT MEASUREMENT

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Abstract. An interferometric method for measuring vibration displacement and acceleration in the 800 Hz – 10 kHz frequency range is described and experimentally tested. Our experimental setup is based on the Twyman-Green interferometer and we were able to measure vibration displacements of 0.12 and 0.27 µm. Depending on frequency, accelerations in the range 1.52 – 10.75 m/s² were obtained.

Key words: interferometry, displacement measurement, vibration analysis, metrology.

1. INTRODUCTION

Interferometry is able to provide traceability of vibration displacement measurement as it relates directly to the definition of meter [1]. The growing need for accurate measurement of vibrations in various applications has led to an internationally accepted standardization regarding procedure and instrumentation of vibration amplitude measurement. In these conditions, the interferometer became a very appealing tool and it was implemented with success for such measurements [2-8].

The most well known interferometric methods to accurately measure and calibrate vibrations [4, 5] are as follows: Fringe Counting Method (FCM) for 1 – 800 Hz, Minimum Point Method (MPM) for 800 Hz – 10 kHz and Sine-Approximation Method (SAM) for 1 Hz – 10 kHz.

Other interferometric methods can visualize vibration modes, measure displacement and phase of vibrations of rough surfaces, such as speckle interferometry [9]. Although, some results on FCM are to be found in [10].
In this work we describe an interferometric setup to measure vibrations using the MPM and we give quantitative and qualitative data regarding vibration displacement measurement in the 800 Hz – 10 kHz frequency range. Displacements of the order of microns are obtained for a full cycle vibration. More advanced interferometric layouts [11] with better resolution make use of quadrature principle to obtain the amplitude of vibrations.

2. THE INTERFEROMETER AND THEORETICAL BACKGROUND

The measuring system is a Twyman-Green interferometer with expanded and collimated beam. As shown in Figure 1, the experimental setup is composed of: He-Ne frequency stabilized laser operating at 632.8 nm (S), beam expander (E), collimating lens (L), non-polarizing beam splitter (BS), reference mirror (M1), measuring mirror (M2), "shaker", signal generator, power amplifier and a computer. A computer records the vibration spectrum via a photo detector (D).

All these components of the measuring system are placed on a vibration isolated optical table. The driving signal for the shaker is controlled by a signal generator and a power amplifier.

When the mirror M2 is subjected to vibrations from the shaker, the Fourier expression of the photo detector output can be written [4] as:

\[ V(t) = V \cos \left[ \phi_0 \left( J_0(\phi_m) - 2J_2(\phi_m) \cos(2\omega t) + 2J_4(\phi_m) \cos(4\omega t) - \ldots \right) \right] - \\
- V \sin \left[ \phi_0 \left( J_1(\phi_m) - 2J_3(\phi_m) \cos(3\omega t) + 2J_5(\phi_m) \cos(5\omega t) - \ldots \right) \right], \tag{1} \]

where \( t \) is time, \( V \) denotes the voltage amplitude, \( \phi_0 \) is the initial phase of the signal, \( \omega \) is the angular frequency \((\omega = 2\pi f, \text{ where } f \text{ is the vibration frequency})\), \( \phi_m \) is the modulation phase amplitude \((\phi_m = 2\pi d/\lambda, \text{ d being the amplitude of the measuring mirror } M_2 \text{ and } \lambda \text{ the wavelength of light})\) and \( J_n \) are the values of the Bessel function.

Fig. 1 – Experimental setup: S – HeNe laser source, E – beam expander, L – collimating lens, BS – beam splitter, M1 – reference mirror, M2 – measuring mirror, D – photo detector, shaker, signal generator, power amplifier and computer.
Thus, with the MPM, one can evaluate the value of the displacement $d$ knowing the argument of the Bessel function that makes the corresponding harmonic zero. It follows that the value of the displacement (e.g. full vibration cycle) can be written as:

$$d = J_{n0} \frac{\lambda}{4\pi},$$  \hspace{1cm} (2)

where $J_{n0}$ is the argument of the Bessel function that generates the $n^{th}$ zero value and $\lambda$ is the wavelength of light used in the interferometer (e.g. 632.8 nm for He-Ne laser).

According to [1-3] the acceleration is expressed as

$$a = \frac{1}{2} J_{n0} f^2 \pi \lambda,$$ \hspace{1cm} (3)

where $f$ is the frequency of vibration.

3. EXPERIMENTAL RESULTS

The 3D model of the experimental setup is shown in Figure 2. Please note that in the actual experiment the interferometer was placed on an optical table and the computer with the signal generator and power amplifier were placed on a different table to reduce unwanted vibrational noise. Thus, the vibration noise floor at the interferometer optical table level in our lab was measured (with a SIOS interferometer) to be approximately 70 ± 5 nm. So, any vibrations with amplitudes greater than this value can be measured with our interferometer.
In this work we chose two frequencies of vibration, namely 0.80 and 0.85 kHz, for which we proposed ourselves to determine the displacement and acceleration of the measuring mirror using the MPM.

For that, the spectrum for each of the two above situations was recorded. Then we minimized the amplitude of vibration in order to reduce the second and respectively the first harmonics of the corresponding vibration motion.

Vibration displacements were computed using equation 2, where $J_{n0}$ is the argument of the Bessel function that generates the $n^{th}$ zero value (e.g. 1st and 2nd harmonics, in our case), where the $n^{th}$ zero value is given when the corresponding harmonic of the vibration frequency is reduced to zero by minimizing the amplitude of vibration.

![Recorded spectra for the vibrations having resonance frequencies of 0.8 and 0.85 kHz. The peaks at 1.6 and 2.4 kHz correspond to the first and the second harmonics of the 0.8 kHz frequency vibration. Respectively, the peaks at 1.7 and 2.55 kHz in the second graph show the harmonics for the 0.85 kHz frequency vibration.](image)

By minimizing one of the harmonics, in each case, we can compute the displacement of the measuring mirror using equation 2, as explained above. Accordingly, the acceleration can be computed, for each case, by using equation 3.

In Fig. 4 the two spectra (e.g. for 0.8 and 0.85 vibration) with the 2$^{nd}$ harmonic reduced to zero are shown. So, both 2.4 kHz and 2.55 kHz peaks are no more visible. At this point we replace $J_{n0}$ with the argument of $J_{20}$ in equations 2 and 3 to find the displacements and accelerations for these two vibration displacements.
Figure 5 shows the spectra of the same vibrations with 2\textsuperscript{nd} and 1\textsuperscript{st} harmonics reduced to zero. Thus, only the resonance frequency peak is visible. Consequently, we use the argument of $J_{10}$ in equations 2 and 3 to compute the displacements and accelerations. In Table 1 we show the resulting displacements and accelerations for the considered vibration displacements. Note that to pass from zero amplitude 2\textsuperscript{nd} harmonic to zero amplitude 1\textsuperscript{st} harmonic a reduction of 0.15 µm in amplitude of vibration is necessary.

### Table 1

<table>
<thead>
<tr>
<th>$f$ [kHz]</th>
<th>$d_{10}$ [µm]</th>
<th>$d_{20}$ [µm]</th>
<th>$a_{10}$ [m/s(^2)]</th>
<th>$a_{20}$ [m/s(^2)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>0.12</td>
<td>0.27</td>
<td>1.52</td>
<td>3.51</td>
</tr>
<tr>
<td>0.85</td>
<td>0.12</td>
<td>0.27</td>
<td>1.72</td>
<td>3.96</td>
</tr>
</tbody>
</table>

$f$ is the vibration frequency; $d_{10}$ is the displacement when the 1\textsuperscript{st} (and implicitly the 2\textsuperscript{nd}) harmonic equals zero; $d_{20}$ is the displacement when the 2\textsuperscript{nd} harmonic equals zero; $a_{10}$ is the acceleration when the 1\textsuperscript{st} (and implicitly the 2\textsuperscript{nd}) harmonic equals zero; $a_{20}$ is the acceleration when the 2\textsuperscript{nd} harmonic equals zero.

Fig. 4 – Recorded spectra for the vibrations having resonance frequencies of 0.8 and 0.85 kHz. The 2\textsuperscript{nd} harmonic peaks do not appear anymore as we reduced them by decreasing the amplitude of vibration.
4. CONCLUSIONS

In this article a method was implemented for vibration displacement measurement at the micrometer scale. The measurement system relies on interferometry and is based on the MPM. Spectra of the studied vibrations showing the resonance frequency and its first and second harmonics are recorded and used to implement the method and compute the displacement vibrations and corresponding accelerations. By reducing the peaks for 2nd and 1st harmonics to zero we were able to compute the above mentioned quantities. We emphasize that, this method is suited for calibrations (e.g. accelerometer calibration) as we are able to compute the amplitude of vibration with certainty only for certain situations (e.g. when making one of the harmonics zero).

The resulting displacements are: 0.12 µm when 2nd harmonic is nulled and 0.27 µm when 1st harmonic is also nulled. Thus, the displacement vibration depend only on amplitude and not on frequency (see also eq. 2). The accelerations vary both with frequency and amplitude and the obtained values are: 1.52 m/s² (for 0.8 kHz and 1st harmonic nulled), 3.51 m/s² (for 0.8 kHz and 2nd harmonic nulled),
1.72 m/s² (for 0.85 kHz and 1st harmonic nulled) and 3.96 m/s² (for 0.85 kHz and 2nd harmonic nulled).

Note that to pass from zero amplitude 2nd harmonic to zero amplitude 1st harmonic a reduction of 0.15 µm in amplitude of vibration is necessary.

REFERENCES