Romanian Reports in Physics, Vol. 62, No. 1, P. 18–26, 2010

NUCLEAR PHYSICS

SINGLE-PARTICLE STATES CLOSE TO THE CONTINUUM IN NEUTRON RICH NUCLEI

R.A. IONESCU, C. HATEGAN

Institute of Atomic Physics, Bucharest, P.O. Box MG-6, Romania

(Received March 18, 2009)

Abstract. We study the energy of single particle states in some light neutron rich nuclei (N=7 and N=9 isotones). From experimental data we extract some general properties of the nuclear mean field and discuss the various effects beyond mean field level which might be responsible for the dependence of the energy on the proton number.

1 INTRODUCTION

The study of the properties of the nuclei near the neutron drip line is one of the most interesting topic in low energy nuclear physics in the last years. For some of these nuclei the existence of a neutron halo, *i.e.* a large spatial extension of the neutron density distribution, is an experimentally well established fact [1, 2]. The existence of borromean nuclei, which are loosely bound systems although the systems with one neutron removed are unbound [2], supports the necessity of a three-body description, the conventional shell-model assumptions being insufficient. The main problem is how to treat the residual neutron-neutron interaction, a perturbative treatment being not able to produce a qualitative change of the spectrum; in other words, one cannot obtain a discrete spectrum in a perturbative treatment if the unperturbed system has only a continuum spectrum.

The results obtained so far allow us to suggest that the most favorable case to observe two neutron halo is the existence of a large negative neutron-core scattering length [3]. This fact is related to the existence of a single-particle s-wave state close to zero energy (a virtual state). The energy of the single-particle states in the nuclear mean field is related to the depth and range of the potential well. Therefore, the study of the dependence of the energy of single-particle states on the number of neutrons and protons is relevant to identify the regions where such new phenomena as nuclear halo might occur.

One assume in the following that the nuclear mean field can be approximated by a spherical square well. Its radius depend only on the total nucleon number, A, as $R = r_0 A^{1/3}$. The depth of the potential well could depend on the relative number of protons and neutrons. The same is true for the strength of the spin-orbit potential which has a surface character; moreover, even a strength of the spin-orbit potential independent of the relative number of protons and neutrons will result in a larger spin-orbit energy splitting for weakly bound state due to their localization at surface.

Let us mention that the virtual state phenomenon does appear for a large domain of light nuclei. For examples two systems displaying *2s* virtual state are ^{9}Li+n and ^{12}Be+n; this is proved by their large negative scattering lengths. One can even speculate that the same *2s* virtual state does manifest over a larger mass-spectrum: this state remains virtual in the system ^{15}B+n. One has to mention other similarities of these three cases: ^{11}Li, ^{14}Be, ^{17}B which are borromean (two-neutron halo) nuclei. This remarks could signify that the evolution of single particle eigenvalues as function on mass number is important for understanding the structure of light nuclei. Specifically, the *2s* neutron state remains near zero-energy in a large range of nuclear masses for appropriate relative number of protons and neutrons.

2. EXPERIMENTAL DEPENDENCE OF $2s_{1/2}$-WAVE, $1p_{1/2}$-WAVE AND $1d_{5/2}$-WAVE SINGLE-PARTICLE ENERGIES

We investigate the experimental dependence of single particle energies on proton number for two chains of isotones nuclei, N = 7 and N = 9 which are relevant for the existence of two neutron halo nuclei ^{11}Li and ^{14}Be.

For N = 7 isotones we consider 2s-wave and $1p_{1/2}$-wave energies. To extract the single particle energies we use the experimental data on neutron separation energies and the experimental energy spectrum. In the spherical mean field picture we can identify the neutron separation energy with the last occupied orbital energy. It might be necessary to include corrections to this simple picture; the main assumption involved is to ignore the difference between single-particle energies in a system with A and a system with $A - 1$ nucleons. This polarization correction could be important for light nuclei, but we expect that its effect can be simulated by a redefinition of the mean field. The excitation energies of other states allow us to define other single-particle energies. The single-particle character of the states is fixed by their spin and parities.

Experimental dependence of s-wave single-particle energy on proton number for N = 7 is showed in Fig. 1. We can see how the first excited state in ^{13}C (the last neutron in the $2s_{1/2}$ single-particle state) evolves smoothly in the ground state of

^{11}Be (the last neutron in the $2s_{1/2}$ single-particle state) and then becomes virtual. The ^{11}Be is known for its abnormal parity in the ground state: the $2s_{1/2}$ single-particle state descends bellow the $1p_{1/2}$ single-particle state for sufficiently small number of protons. Let us remark that we obtained the energy of $2s_{1/2}$ single-particle state in the odd-odd nucleus ^{12}B from the mean value of the energies of the second and third excited states which have negative parities and angular momenta 2 and 1; these are consistent with the last neutron in a $s_{1/2}$ orbital and the last proton (the 5th one) in a $p_{3/2}$ orbital.

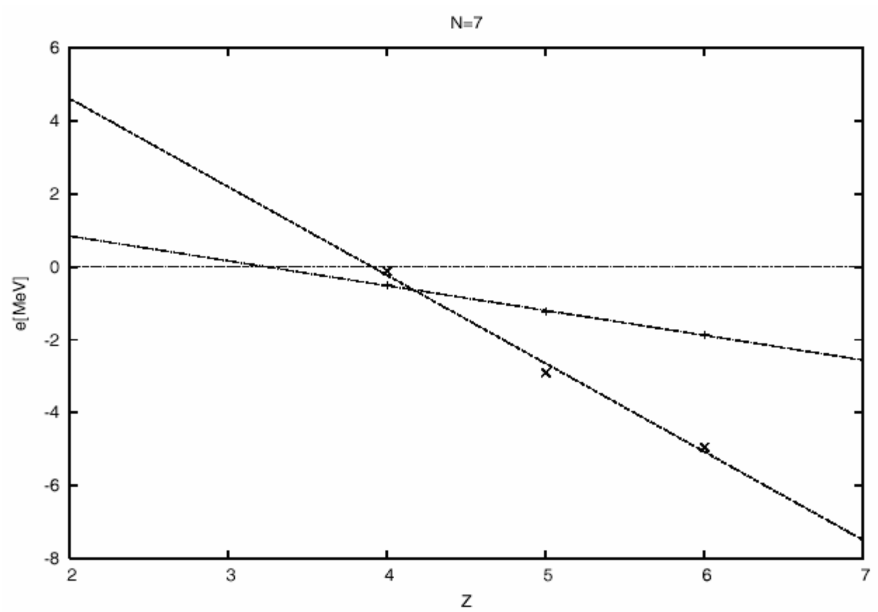

Fig. 1 – Experimental dependence of *s*-wave (dotted line) and $p_{1/2}$-wave (dashed line) single particle energy on proton number for N=7 isotones.

The results for $1p_{1/2}$ single-particle states are showed in Fig. 1 together with the results for $2s_{1/2}$ single-particle states. The energy of the $1p_{1/2}$ single-particle state in the odd-odd nucleus ^{12}B is obtained from the mean value of the energies of the ground and the first excited states which have positive parities and angular momenta 1 and 2; these are consistent with the last neutron in a $p_{1/2}$ orbital and the last proton (the 5th one) in a $p_{3/2}$ orbital.

We see the same smooth behaviour of the energy when the number of protons decreases, the slope of the curve being smaller for *s*-wave states then that for *p*-wave states.

In Fig. 2 we plotted also the experimental dependence of *s*-wave single-particle energy on proton number for $N = 9$. We can see how the first excited state in ^{17}O (the last neutron in the $2s_{1/2}$ single-particle state) evolves smoothly in the ground state of ^{15}C (the last neutron in the $2s_{1/2}$ single-particle state) and then becomes virtual. Let us remark that we obtained the energy of $2s_{1/2}$ single-particle state in the odd-odd nucleus ^{16}N from the mean value of the energies of the first and third excited states which have negative parities and angular momenta 0 and 1; these are consistent with the last neutron in a $s_{1/2}$ orbital and the last proton (the 7^{th} one) in a $p_{1/2}$ orbital. The energy of $2s_{1/2}$ single-particle state in the odd-odd nucleus ^{14}B was obtained as the mean value of the energies of the ground state and the first excited state which have negative parities and angular momenta 2 and 1; these are consistent with the last neutron in a $s_{1/2}$ orbital and the last proton (the 5^{th} one) in a $p_{3/2}$ orbital.

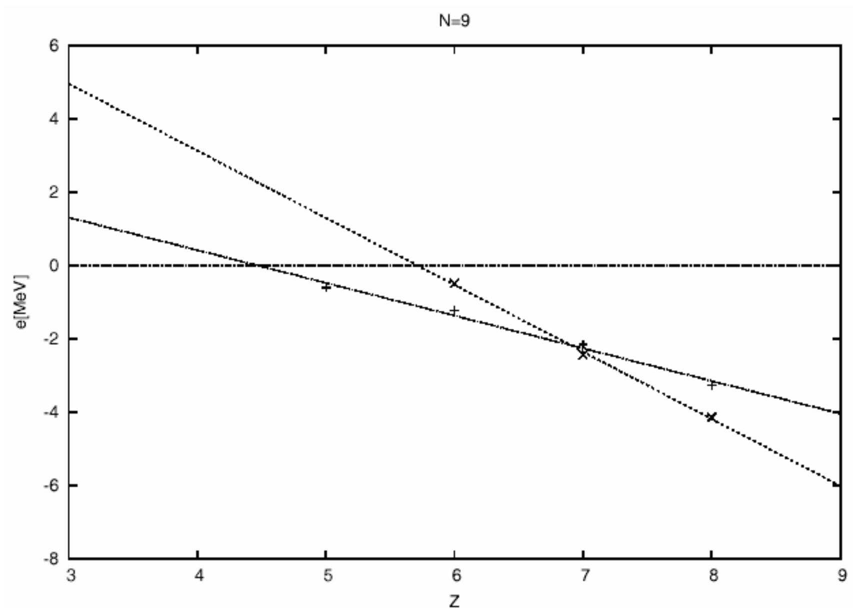

Fig. 2 – Experimental dependence of s-wave (dotted line) and $d_{5/2}$-wave (dashed line) single particle energy on proton number for N=9 isotones.

The results for $1d_{5/2}$ single-particle states are showed in Fig. 2 together with the results for $2s_{1/2}$ single-particle states. The same smooth behaviour of the energy when the number of protons decreases is observed, the slope of the curve being smaller for s-wave states then that for d-wave states.

We see that the behaviour of the neutron $2s_{1/2}$ single-particle state is similar for N = 7 and N = 9 isotones. The slope is larger for N = 9 and the zero energy intercept is at larger Z for N = 9.

In the next section we will investigate theoretically the properties of the nuclear mean field responsible for this behaviour.

3. ENERGIES OF SINGLE-PARTICLE STATES IN A POTENTIAL WELL

One assume in the following that the nuclear mean field can be approximated by a spherical square well. Its radius depend only on the total nucleon number, A, as $R = r_0 A^{1/3}$ and we take $r_0 = 1.2\ fm$. The depth of the potential well could depend on the relative number of protons and neutrons and we did calculate numerically the energies of single-particle states for different depths independent of the relative number of neutrons and protons as well as for depths that depends on it. We started with a mean field

$$V = -V_s + V_v \frac{N-Z}{N+Z},$$

where V_s = 50MeV and V_v = 33MeV [4]. This mean field acting on neutrons decreases with decreasing proton number for neutron rich nuclei.

The numerical algorithm is based on the equation which gives the energies at which the logarithmic derivative of the wave function inside the well is equal with the logarithmic derivative of the wave function outside the well.

Inside the potential well the radial wave function for orbital angular momentum zero is

$$\Re_0 = \frac{\sin kr}{r},$$

where $k = \frac{1}{\hbar}\sqrt{2m\left(U - |E|\right)}$ with $U = -V > 0$ the depth of the attractive potential and E the single energy, m being the neutron effective mass. The radial wave function for orbital angular momentum one is

$$\Re_1 = \frac{\sin kr}{kr^2} - \frac{\cos kr}{r}.$$

The radial wave function for orbital angular momentum two is

$$\Re_2 = \frac{\left(3 - k^2 r^2\right)\sin kr}{kr^3} - \frac{3\cos kr}{r^2}.$$

For higher values of orbital angular momentum the radial wave functions are given in terms of higher order spherical Bessel function regular in origin. The above forms of the radial wave functions results into the following logarithmic derivatives, L_l at boundary, R

$$RL_0^{\text{int}} = \frac{kR}{\tan kR},$$

$$RL_1^{\text{int}} = \frac{\left[(kR)^2 - 1\right]\sin kR + kR\cos kR}{\sin kR - kR\cos kR},$$

$$RL_2^{\text{int}} = \frac{\left[3(kR)^2 - 6\right]\sin kR + kR\left[6 - (kR)^2\right]\cos kR}{\left[3 - (kR)^2\right]\sin kR - 3kR\cos kR}.$$

Outside the potential well the radial wave functions that vanish at infinity are

$$\Re_l = \text{const.}(i)^l \frac{r^l}{K^l}\left(\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\right)^l \frac{\mathrm{e}^{-Kr}}{r},$$

where $K = \frac{1}{\hbar}\sqrt{2m|E|}$. These radial wave functions results into the following logarithmic derivatives at boundary

$$RL_0^{ext} = -KR,$$

$$RL_1^{ext} = -KR - \frac{1}{1+KR},$$

$$RL_2^{ext} = -KR - \frac{3KR+6}{(KR)^2 + 3KR + 3}.$$

The equations $L_l^{int} = L_l^{ext}$ are solved numerically for $-U < E < 0$. The spin-orbit interaction was included as a delta shell potential with the standard form $f(r)\vec{l}\vec{s},\quad f(r) = \lambda\frac{1}{r}\frac{\mathrm{d}V}{\mathrm{d}r}$ and $\lambda = -0.5 fm^2$ [5]. With the standard parametrization of the nuclear mean field [4] we obtained no 2*s*-wave bound state for N = 7 isotones with Z = 4,5,6 and an overbinding of the order of 5 – 8MeV for the $1p_{1/2}$ single particle state. One can obtain the experimental single particle energies for 2*s* state if we use a larger strength of the nuclear mean field for *s* states; this square-well potential should be of the order of 65MeV and should increase slightly with decreasing proton number, contrary to the standard parametrization. This depth is 30%–60% larger than the standard depth. The experimental $1p_{1/2}$ single particle energies are obtained with a square-well potential with a smaller depth, but with a standard dependence of this depth on the neutron and proton numbers. This depth is 20% smaller than the standard depth.

For N = 9 isotones we obtained with the standard parametrization of the mean field [4] a ~ 3MeV underbinding of the 2*s* state for Z = 8, no 2*s* bound states for Z = 5, 6, 7 and an overbinding of the order of 1-3MeV for the $1d_{5/2}$ single particle state. One can obtain the experimental single particle energies for 2*s* state if we use a larger strength of the nuclear mean field for *s* states similar to the case of $N = 7$ isotones; this square well potential should be of the order of 55MeV and should increase slightly (more slightly than in the $N = 7$ isotones chain) with decreasing proton number, contrary to the standard parametrization. This depth is 15%–40% larger than the standard depth.

The experimental $1d_{5/2}$ single particle energies are obtained with a square-well potential with a smaller depth, but with a standard dependence of this depth on the neutron and proton numbers. This depth is 6% smaller than the standard depth.

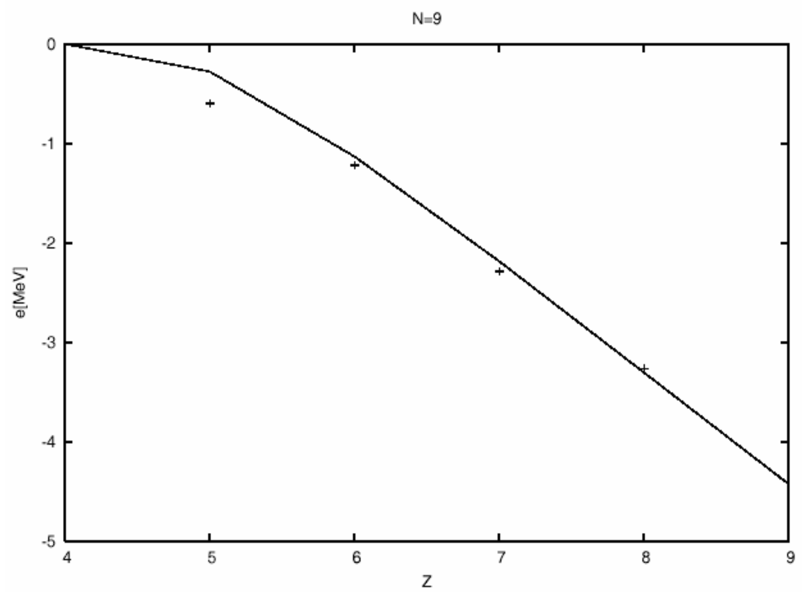

Fig. 3 – Experimental (points) and theoretical (line) dependence of *s*-wave single particle energy on proton number for N=9 isotones.

The results obtained for 2*s*-wave states are plotted in Fig. 3 together with the experimental values for $N = 9$ isotones with a depth $U = 55$MeV independent on the relative number of protons and neutrons. Of course, the zero energy intercept depends only on the product UR^2. The position of zero energy 2*s*-wave state depends on the potential depth as well as on its dependence on relative number of protons and neutrons, but the numerical calculations shows that the slope of the dependence of the energy on proton number around zero energy intercept depends only on the dependence of the depth on the relative neutron-proton number: for usual dependence [4] when the mean field acting on neutrons decreases with decreasing proton number for neutron rich nuclei the slope is too large compared with the experimental values. A similar description for $N = 7$ isotones is obtained with a larger potential depth, $U = 65$MeV and a mean field which increases with decreasing proton number.

For the $N = 7$ isotones the slope of the dependence of the energy on proton number around zero energy intercept for $1p_{1/2}$ states depends only on the dependence of the depth on the relative neutron-proton number and not on the depth itself or the particular value of the spin-orbit interaction. From our calculations we can conclude that the experimental dependence is consistent with a mean field independent on the relative number of protons and neutrons; in such a case the different slopes for *s*- and *p*-wave energies are correctly reproduced but the depth of the potential should be smaller for *p*-wave states to get the experimental observed anomal parity of the ground state in ^{11}Be.

4. DISCUSSIONS AND CONCLUSIONS

From our investigation we can conclude that the slope of the energy dependence of the single particle states on the proton number around zero energy for light neutron rich nuclei is consistent with a nuclear mean field acting on neutrons independent on the relative number of protons and neutrons in the nuclear system. On the other hand from the study of nuclei close to the β stability line it was extracted a dependence of the mean field on the relative number of protons and neutrons. This discrepancy could have two causes: the extrapolation of mean field parametrizations established in the study of stable nuclei toward neutron-rich nuclei is unreliable or the effects of residual interactions are more important for weakly bound nuclear systems. The answer to this question needs further investigations.

The different slopes of *s*-wave energies for $N = 7$ and $N = 9$ isotones are related to the difference in the dependence of the potential depth with respect to the relative number of protons and neutrons in the two cases. The physical reason for that particular dependence should be investigated more closely.

We should mention two effects which might affect the values of the mean field for light neutron rich nuclei. The mean field might be different for neutron *s* and *p* states due to the single-particle structure of the core: more *p*-wave protons could increase the mean field acting on the *p* wave neutrons. Also, different ground state deformation of neutron and proton density distributions are expected in some nuclei with extreme isospin projection quantum number, T_z, *i.e.* for large difference between neutron and proton number.

In conclusion we can say that the most important fact is the increasing of the nuclear mean field for *s* states for neutron rich nuclei. This might be due to new modes of core polarization when there is a large difference in the neutron and proton numbers.

Acknowledgement. This work was partially supported by the Ministry of Education and Research (Contract IDEI 43/2007).

REFERENCES

1. K. Riisager, Rev. Mod. Phys., **66**, 1105 (1994).
2. M.V. Zhukov, B.V. Danilin, D.V. Fedorov, J.M. Bang, I.J. Thompson, J.S. Vaagen, Phys. Rep., **231**, 151(1993).
3. R.A. Ionescu, C. Hategan, Europhys. Lett., **45**, 296(1999).
4. A. Bohr, B. Mottelson, *Nuclear Structure*, Vol. I, W.A. Benjamin, New York, 1975.
5. P. Ring, P. Schuck, *The Nuclear Many-Body Problem*, Springer Verlag, New York, 1980.