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QUANTUM ASPECTS OF PHOTON PROPAGATION IN TRANSPARENT INFINITE HOMOGENEOUS MEDIA

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Abstract. The energy balance photon – medium, during the light travelling, through a specific continuous interaction between a single photon and a homogeneous, infinite medium (fully ionized plasma or a transparent dielectric), was studied. We obtained a wave equation for the interacting photon. To explain the interaction in quantum terms, we assume a certain photon – medium interaction energy, macroscopically materialized by the existence of the refractive index. It turns out that the interaction is of a scalar type, for vanishing rest mass and of spin 1 particle submitted both to scalar and vectorial fields. We found out an expression of the propagation equation of the photon through a non-dissipative medium, using a coupling between the photon spin $\vec{S}$ and the scalar interaction field $(\vec{E}_S, \vec{H}_S)$.

Key words: Photon – medium interaction, photon spin - scalar field coupling, refraction index.

1. INTRODUCTION. KRAMERS TYPE FORMALISM

Many years ago, Nicholas Ionesco-Pallas proposed [1] a new formulation of Electrodynamics in homogeneous moving media, different from those already known, due to Abraham and Minkowski. See for instance [2]. This new approach is based on two distinct hexa-vectors $\vec{Q}_A = \left(\vec{E}, \vec{B}\right)$ and $\vec{Q}_B = \left(\vec{D}, \vec{H}\right)$, or two Q-vectors [3, 4].

In Romanian literature, Q-vectorial formulation of Electrodynamics was promoted by Nicholas Ionesco-Pallas *et al.* [5–7] and George Moisil [8]. In view of ensuring a correct relativistic behavior of time relaxation in dielectrics, a certain constraint is, however, necessary to be accomplished before any application of the Q-vectorial formalism. This is a connection between the resistivity as a 4-vector, ξ^a, and the conductivity λ under the form

$$\xi^a = \left(-\frac{i}{\lambda(\vec{v})}\frac{\vec{v}}{c}, \frac{1}{\lambda(\vec{v})}\right). \tag{1}$$

The constraint is equivalent to the statement that "The structure of Electrodynamics in moving homogeneous media may be organized in terms of *Q*-vectors if, and only if, conductibility and resistivity are combined together as a 4-vector". In other words, conductibility must vary, in terms of velocity as

$$\lambda(\vec{v}) = \lambda(0)\sqrt{1-\frac{\vec{v}^2}{c^2}} \tag{2}$$

In this paper, the effort of the author is focused on a rather different direction, than it is usually paid by the physicists working with photons. While the great part of them deals with free photons implied in optical experiments or confined in ideal cavities and constrained to thermal equilibrium [9–13], the purpose of this paper is to study the behavior of photons, interacting with the environmental matter, during the path of light through plasma or dielectric media. The special character of this interaction is coming from the physical characteristics of the photon – its inertial mass and its electrical charge are vanishing. A methodological principle may, however, be useful in this case, namely an analogy between the light propagation in an optical guide and the light propagation in plasma near the cut off limit [14, 15]. Additional information may be equally gathered when classical Fresnel's formulas are reinterpreted in modern quantum mechanical terms due to an idea of W. Heisenberg [18–20]. Finally, a surprising idea is coming from the possibility to put into evidence even the coupling of the photon spin with the gradient of the interacting energy in propagation through a dielectric medium. All these aspects deserve a unitary treatment in a revised approach and make the subject of the present work. At the same time, arguments are brought to accept the idea that the wave function of the interacting photon is a *Q*-vector (connected in a specific way with an irreducible spinor).

As far as we are convinced that Relativity Theory is a theoretical tool for entering upon any problem of modern physics (as a result of several already worked out experiments), we have no doubt that conductibility variation in terms of velocity will be confirmed too.

2. THE PHOTON WAVE FUNCTION

So, coming back to our problem (which is a quantum one) we may state that two *Q*-vectors $\vec{Q}_A$ and $\vec{Q}_B$ are necessary for a complete description of energy propagation in this case, namely

$$\vec{Q}_A = \vec{E} + i\vec{B}, \quad \vec{Q}_B = \vec{D} + i\vec{H}. \tag{3}$$

The de Broglie wave propagation associated to the photon propagation is written as

$$\vec{\Psi} = K_A \vec{Q}_A + K_B \vec{Q}_B . \tag{4}$$

For a homogeneous and transparent medium with

$$\vec{D} = \varepsilon \vec{E}, \quad \vec{B} = \mu \vec{H}, \quad n = \sqrt{\varepsilon\mu} > 1, \tag{5}$$

one obtains

$$\vec{\Psi}^* \vec{\Psi} = (K_A + \varepsilon K_B)^2 \vec{E}^2 + (K_B + \mu K_A)^2 \vec{H}^2 \equiv |C|^2 \frac{1}{8\pi} (\varepsilon \vec{E}^2 + \mu \vec{H}^2). \tag{6}$$

Here $|C|^2$ is normalization constant. Identifying the constant in (6), one obtains the algebraic system of equations.

$$\begin{aligned} K_A + \varepsilon K_B &= |C| \frac{1}{\sqrt{8\pi}} \sqrt{\varepsilon}, \\ K_B + \mu K_B &= |C| \frac{1}{\sqrt{8\pi}} \sqrt{\mu}, \end{aligned} \tag{7}$$

whose solution is

$$K_A = |C| \frac{1}{\sqrt{8\pi}} \frac{\sqrt{\varepsilon}}{n+1}, \qquad K_B = |C| \frac{1}{\sqrt{8\pi}} \frac{\sqrt{\mu}}{n+1}. \tag{8}$$

The photon wave function is now normalized in such a way so that the presence probability of a photon is proportional to the energy density in the Euclidian space.

$$\vec{\Psi} = |C| \frac{1}{\sqrt{8\pi}} \left(\frac{\sqrt{\varepsilon}}{n+1} \vec{Q}_A + \frac{\sqrt{\mu}}{n+1} \vec{Q}_B \right) = |C| \frac{1}{\sqrt{8\pi}} \left(\sqrt{\varepsilon} \vec{E} + i \sqrt{\mu} \vec{H} \right), \tag{9}$$

$$\int \vec{\Psi}^* \vec{\Psi} dx_1 dx_2 dx_3 = |C|^2 \int \frac{\vec{E} \cdot \vec{D} + \vec{H} \cdot \vec{B}}{8\pi} dx_1 dx_2 dx_3 . \tag{10}$$

3. THE MATRIX REPRESENTATION OF PHOTON WAVE FUNCTION

It is now convenient to adopt a matrix representation, alternatively to the standard vectorial representation:

$$\vec{\Psi} \to |C| \frac{1}{\sqrt{8\pi}} \begin{bmatrix} \sqrt{\varepsilon} E_x + i \sqrt{\mu} H_x \\ \sqrt{\varepsilon} E_y + i \sqrt{\mu} H_y \\ \sqrt{\varepsilon} E_z + i \sqrt{\mu} H_z \end{bmatrix} \equiv \begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \end{bmatrix}. \tag{11}$$

For every component Ψ_k there is a propagation equation

$$\left(\Delta-\frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\psi_k=0,\qquad \psi_k=\psi_k(0)e^{i(\vec{k}\vec{r}-\omega t)},\qquad v=\frac{c}{\sqrt{\varepsilon\mu}}. \tag{12}$$

The momentum and energy operators may operate delivering the dispersion equation

$$\begin{aligned}&\hat{\vec{p}}=\frac{\hbar}{i}\Delta,\qquad \hat{E}=-\frac{\hbar}{i}\frac{\partial}{\partial t}\ \Rightarrow\\ &\hat{\vec{p}}\Psi_k=\hbar\vec{K}\Psi_k,\qquad \hat{E}\Psi_k=\hbar\omega\Psi_k,\\ &\left(\vec{K}^2-\frac{\omega^2}{v^2}\right)\Psi_k=0,\qquad \to|\vec{K}|=\frac{\omega}{v}=\frac{\omega}{c}\sqrt{\varepsilon\mu}.\end{aligned} \tag{13}$$

The recovering of the dispersion $(|K|,\omega)$ relationship is a proof for using momentum operators $(\vec{p},\hat{E})$ just as in Quantum mechanics.

On the other hand, the Maxwell equations in a transparent infinite homogeneous medium can be written in the form

$$\begin{aligned}&\nabla\left(\sqrt{\varepsilon}\vec{E}\right)=0,\qquad \nabla\times\left(\sqrt{\varepsilon}\vec{E}\right)=-\frac{\sqrt{\varepsilon\mu}}{c}\frac{\partial}{\partial t}\left(\sqrt{\mu}\vec{H}\right),\\ &\nabla\left(\sqrt{\mu}\vec{H}\right)=0,\qquad \nabla\times\left(\sqrt{\varepsilon}\vec{E}\right)=-\frac{\sqrt{\varepsilon\mu}}{c}\frac{\partial}{\partial t}\left(\sqrt{\varepsilon}\vec{E}\right).\end{aligned} \tag{14}$$

The curl-type equation of (14) can be further written in the form of a single equation for the complex function $\vec{\Psi}$ as defined in (9), so that

$$\begin{aligned}&\nabla\times\vec{\Psi}=\frac{i}{v}\frac{\partial\vec{\Psi}}{\partial t},\qquad \nabla\cdot\vec{\Psi}=0,\\ &v=\frac{c}{\sqrt{\varepsilon\mu}}.\end{aligned} \tag{15}$$

Actually, the first equation (15) is a Schrödinger-type equation. In order to make this assertion more convincing, let us introduce the energy operator from (13), the matrix notation of $\vec{\Psi}$ from (11) and the spin 1 algebra, namely

$$\vec{\mathcal{S}}\times\vec{\mathcal{S}}=i\vec{\mathcal{S}},\quad \vec{\mathcal{S}}^{+}=\vec{\mathcal{S}}, \tag{16}$$

$$\vec{\mathcal{S}}=\begin{bmatrix}0&0&0\\0&0&-i\\0&i&0\end{bmatrix}\vec{i}_1+\begin{bmatrix}0&0&i\\0&0&0\\-i&0&0\end{bmatrix}\vec{i}_2+\begin{bmatrix}0&-i&0\\i&0&0\\0&0&0\end{bmatrix}\vec{i}_3. \tag{17}$$

We get in this way a perfect equivalence between the matrix and vector notations, that is

$$\vec{\Psi} = \Psi_1 \vec{i}_1 + \Psi_2 \vec{i}_2 + \Psi_3 \vec{i}_3, \qquad \vec{\Psi}^* = \Psi_1^* \vec{i}_1 + \Psi_2^* \vec{i}_2 + \Psi_3^* \vec{i}_3,$$

$$\Psi = \begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \end{bmatrix}, \qquad \Psi^+ = [\Psi_1^* \quad \Psi_2^* \quad \Psi_3^*], \tag{18}$$

$$\vec{\Psi}^* \cdot \vec{\Psi} = \sum\nolimits_{jk} \delta^{jk} \Psi_k^* \Psi_j = \sum_{k=1}^{k=3} \Psi_k^* \Psi_k = \Psi^+ \Psi.$$

Also

$$\left(\vec{S} \cdot \vec{a}\right)\Psi = i \begin{bmatrix} 0 & -a_3 & a_2 \\ -a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \end{bmatrix} = i \begin{bmatrix} a_2 \Psi_3 - a_3 \Psi_2 \\ a_3 \Psi_1 - a_1 \Psi_3 \\ a_1 \Psi_2 - a_2 \Psi_1 \end{bmatrix} = i \begin{bmatrix} (\vec{a} \times \vec{\Psi})_1 \\ (\vec{a} \times \vec{\Psi})_2 \\ (\vec{a} \times \vec{\Psi})_3 \end{bmatrix}$$

$$\Rightarrow \qquad \begin{aligned} &\left(\vec{S} \cdot \vec{a}\right)\Psi = i\left(\vec{a} \times \vec{\Psi}\right), \\ &\left(\vec{S} \cdot \hat{\vec{p}}\right)\Psi = \hbar\left(\nabla \times \vec{\Psi}\right). \end{aligned} \tag{19}$$

4. THE PHOTON WAVE EQUATION

As a consequence of the last equalities, the first equation (15) takes the specific form

$$\hat{E}\Psi = \hat{H}\Psi, \quad \hat{E} = -\frac{\hbar}{i}\frac{\partial}{\partial t}, \quad \hat{H} = v\left(\vec{S} \cdot \vec{p}\right). \tag{20}$$

In our opinion, this is the photon wave equation for an infinite medium characterized by the constants (ε, μ). Quite specific for this equation is the "irrelevance" of the constant $\hbar$. From Eq. (18) we get

$$-\frac{\hbar}{i}\frac{\partial \vec{\Psi}}{\partial t} = v\hbar\left(\nabla \times \vec{\Psi}\right), \qquad +\frac{\hbar}{i}\frac{\partial \vec{\Psi}^*}{\partial t} = v\hbar\left(\nabla \times \vec{\Psi}^*\right). \tag{21}$$

By the scalar multiplication at left of the first of these equations by $\vec{\Psi}^*$ and, similarly, of the second one by $\vec{\Psi}$, and subtracting the results, we finally get

$$-\frac{\hbar}{i}\left(\vec{\Psi}^* \cdot \frac{\partial \vec{\Psi}}{\partial t} + \frac{\partial \vec{\Psi}^*}{\partial t} \cdot \vec{\Psi}\right) = v\hbar\left\{\vec{\Psi}^* \cdot \left(\nabla \times \vec{\Psi}\right) - \vec{\Psi} \cdot \left(\nabla \times \vec{\Psi}^*\right)\right\}. \tag{22}$$

Using further the identity

$$\vec{B}\cdot(\nabla\times\vec{A})-\vec{A}\cdot(\nabla\times\vec{B})\equiv\nabla\cdot(\vec{A}\times\vec{B}) \tag{23}$$

for $\vec{B}=\vec{\Psi}^*$ and $\vec{A}=\vec{\Psi}$ we conclude that the right part of Eq. (22) is reduced to the compact form $v\hbar\nabla\cdot(\vec{\Psi}\times\vec{\Psi}^*)$ while the left part can be reduced to $-\frac{\hbar}{i}\frac{\partial}{\partial t}(\vec{\Psi}^*\cdot\vec{\Psi})$.

In this way equation (22) becomes

$$\frac{\partial}{\partial t}(\vec{\Psi}^*\cdot\vec{\Psi})+\nabla\cdot\{iv(\vec{\Psi}\times\vec{\Psi}^*)\}=0. \tag{24}$$

In addition, it can easily be verified the following identities

$$\begin{aligned}&\vec{\Psi}^*\cdot\vec{\Psi}=\Psi^+\cdot\Psi,\\&\Psi^+\vec{S}\Psi=i(\Psi_3^*\Psi_2-\Psi_2^*\Psi_3)\vec{i}_1+i(\Psi_1^*\Psi_3-\Psi_3^*\Psi_1)\vec{i}_2+i(\Psi_2^*\Psi_1-\Psi_1^*\Psi_2)\vec{i}_3,\\&\Psi^+\vec{S}\Psi=i(\vec{\Psi}\times\vec{\Psi}^*),\end{aligned} \tag{25}$$

so that the equation (24) gets the "quantum" form

$$\frac{\partial}{\partial t}(\Psi^+\Psi)+\nabla\cdot\{v\psi^+\vec{S}\Psi\}=0. \tag{26}$$

Obviously, this equation allows simultaneously two interpretations:

a) as the conservation law of the photon localization probability, and
b) as the conservation law of the corresponding electromagnetic energy.

The two interpretations are interrelated and allow the determination of the normalization constant $|C|$.

Let's now go back to Eq. (12) in its quasi-d'Alambertian form

$$\Delta\Psi-\frac{1}{v^2}\frac{\partial^2}{\partial t^2}\Psi=0,\qquad v=\frac{c}{\sqrt{\varepsilon\mu}}. \tag{27}$$

The immediate temptation is to "generalize" this equation for an arbitrary $v\neq 0$. For this purpose, we add and subtract the term $-\frac{1}{v^2}\frac{\partial^2}{\partial t^2}\Psi$ so that we compose the Lorentz operator as a d'Alambert invariant, namely

$$\left(\Delta-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Psi+\frac{1}{c^2}(1-n^2)\frac{\partial^2\Psi}{\partial t^2}=0. \tag{28}$$

On the other hand, from Eq. (13) it results that the action of the operator $\frac{\partial}{\partial t}$ upon the wave function Ψ is

$$\frac{\partial}{\partial t} = -\frac{i}{h}\hat{E} \to -i\omega, \tag{29}$$

so that from Eqs. (28) and (29) we finally get

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Psi - \left(\frac{\omega}{c}\right)^2 (1-n^2)\Psi = 0. \tag{30}$$

So far, the photon medium interaction was investigated under its energetic aspect, and the final equation (30) is equivalent to Helmholtz equation (27) . Possible additional interactions, coming from the coupling of the photon spin with the gradient of the medium potential energy U, (if any), were assumed quite small and disregarded. In this first aproximation, the wave function Ψ is a scalar quantity, and the probabilitie interpretation on Ψ reduces to the additional integrability condition of $\Psi^*\Psi$ over the three – dimensional Euclidian space E_3.

To go further in the photon-medium interaction procces, we need to know more about the photon spin. There is a certain well-defined relationship betwen the dimensionality of the Hilbert space wave functions and spin. For photon, the respective dimensionality is $2S+1=3$, or, if we want to account for the parity conservation too, this number may be dubled $2(2S+1)=6$. We observe that the dimensionality 4 is prohibited for photon. The nature of this prohibition was enlightned in the paper „Entering upon the spinor concept from a vectorial stand point“ – work due to Nicholas Ionesco-Pallas [24] and published in 1970. The mentioned author demonstrated that the wave equation, in the case when Ψ is a Minkowski 4 – vector does separate – via a Foldy transformation for avoiding the negative energy states – into a scalar equation and a spin 1 equation (having three wave components). This is in strong contrast with the wide (and groundless) opinion that the photon wave function is delivered by the 4 – potential equation.

5. THE PHOTON PROPAGATION THROUGH A DIELECTRIC MEDIUM

Let's continue with a reasoning based on experiment as described in [14, 15]. In this case the propagation medium of the photon is represented by infinite non-dispersive plasma (Fig. 1), hence the wave equation of the photon can be written in the form

$$\begin{aligned} &\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Psi - K^2\Psi = 0, \\ &K \equiv \frac{\omega}{c}\sqrt{1-n^2}. \end{aligned} \tag{31}$$

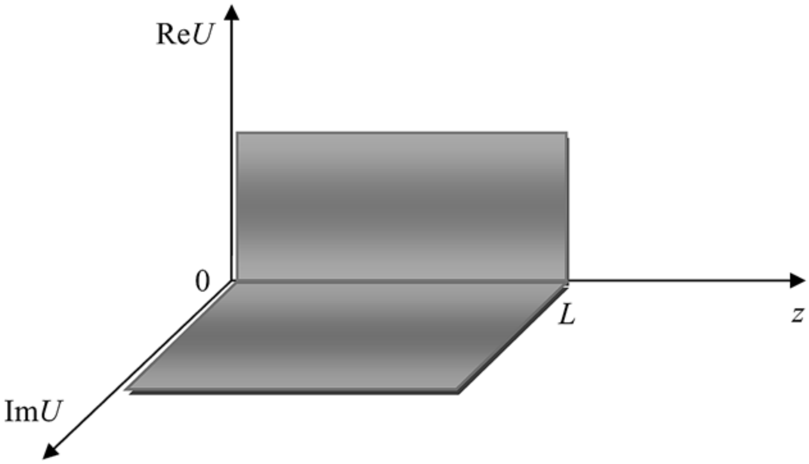

Fig. 1 – The complex space of the potential energy of interaction between photons and the propagation medium. L is the dielectric length.

Also in [14] has been introduced the notation

$$U = U_p = c\hbar K = \hbar\omega\sqrt{1-n^2}, \quad n<1, \tag{32}$$

where the quantity U_p denotes the medium (plasma) potential, so that the dispersion equation of the photon in the plasma is

$$\hbar\omega = \left(U_p^2 + c^2\vec{p}^2\right)^{\frac{1}{2}}. \tag{33}$$

Let's consider further the propagation of a photon through a dielectric medium. For this purpose we will start from the energy equation of a particle moving in a field composed of both a vectorial component (of the electrodynamic type) and a complex scalar component (of the meso-dynamic type). An argument in favor of the scalar nature of the interaction between a photon and the dielectric medium resides in allowing the use of Fermat's principle for the propagation of a light ray just as in Einstein's theory of gravitation. In conclusion, a particle moving in a field characterized by both vector potential $\vec{A}$ and scalar potential Φ, as well as by a scalar potential U, has the following expression

$$\begin{aligned} &E = q\Phi + \left[(m_0c^2+U)^2 + (c\vec{p}-q\vec{A})^2\right]^{\frac{1}{2}}, \\ &U = \mathrm{Re}U + i\,\mathrm{Im}U. \end{aligned} \tag{34}$$

Here the quantity U has the dimensions of energy and not of a potential as has been denominated in [14, 15]. Consequently, U is not comparable with Φ (the electric scalar potential) or with $\vec{A}$ (the magnetic vector-potential) but with $q\Phi$, respectively with $q\vec{A}$. In conclusion, even in the limiting case $m_0 \to 0$ (when the particle becomes a photon and has no electrical charge) a particle can carry a "charge", q_S, of non-electromagnetic nature, as a result of the permanent interaction of the photon with the medium through which it propagates. In other words, we can formally write $U = q_S\Phi_S$, (with the index S from the word scalar). However, the separation of the quantity U into the product charge X potential = interaction energy, possible in the case of a mesonic interaction, do not allow the univocal definition of a scalar charge characterizing a local interaction of the photon with the propagation medium (transparent dielectric or plasma). The propagation is non-dispersive if ImE = 0, that is (for m_0 = 0, q = 0)

$$\mathrm{Re}U \cdot \mathrm{Im}U = 0. \tag{35}$$

Consequently, from (34) and (35) we get

$$\begin{aligned} &E = \left[(\mathrm{Re}U)^2 - (\mathrm{Im}U)^2 + c^2\vec{p}^2\right]^{\frac{1}{2}}, \\ &\mathrm{Re}U \cdot \mathrm{Im}U = 0, \quad m_0 = 0, \quad q = 0. \end{aligned} \tag{36}$$

We have from here two distinct situations for the photon non-dispersive propagation through an infinite medium (Fig. 1):

$$\begin{cases} a)\ \mathrm{Im}U = 0, \quad U = U_p = \hbar\omega\sqrt{1-n^2}, \quad n<1 \quad \text{(plasma medium)} \\ b)\ \mathrm{Re}U = 0, \quad U = U_d = i\hbar\omega\sqrt{n^2-1}, \quad n>1 \quad \text{(dielectric medium).} \end{cases}$$

It follows that the equation of photon propagation through an infinite dielectric transparent medium, equivalent with Eq. (33) of photon propagation through a plasma, is

$$\hbar\omega = \left(U_d^2 + c^2\vec{p}^2\right)^{\frac{1}{2}}, \qquad U = U_d = i\hbar\omega\sqrt{n^2-1}, \quad n>1. \tag{37}$$

Summarizing, notwithstanding the propagation medium, the photon wave equation is

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Psi + \alpha K^2 \Psi = 0, \tag{38}$$

$$K = \frac{\omega}{c}\left|n^2 - 1\right|^{\frac{1}{2}}, \qquad \alpha = \begin{cases} -1 & \text{(plasma)} \\ +1 & \text{(dielectric).} \end{cases}$$

For $\alpha = -1$ the static solution of Ψ is of the Seelinger-Mie-Proca-Yukawa type, and for $\alpha = +1$ it is of the Helmholtz type.

The use of equation (38) allows the description of the passage of light from a medium with refraction index $n_0 \approx 1$ into a medium with refractive index $n > 1$. In addition, at the separation surface of the two media one considers the laws of conservation of photon energy and impulse. Also the normalization of the photon wave function is achieved by fixing the steady flux of incident photons. Under these conditions, the refractive index plays the same role for the photon just as it does the electrical potential for a charged particle.

A particular case of potential pointed out in [14, 15], is that one which is generated by the guided propagation of photons for which we distinguish the following situations:

a) Empty wave guide. The spatial limitation exerted by the wave guide walls perpendicularly to the propagation direction generates the potential energy U_g of the form

$$U_g = \hbar\omega_{critic}, \tag{39}$$

where ω_{critic} is the critical frequency of the wave guide. This is a real potential energy.

b) Lossless dielectric filled wave guide. The rule of energy summation is applied in the form

$$U_{g+d} = \sqrt{U_g^2 + U_d^2}, \tag{40}$$

U_d being imaginary, yet the quantity under root in (40) can be either positive or negative; also the potential energy U_{g+d} can be either real and smaller than the potential energy of empty wave guide, or alternately, it can be imaginary.

c) Lossless plasma filled guide (Fig. 2). In this case the resulting potential U_{g+p} is always real (see [14, 15]).

$$U_{g+p} = \sqrt{U_g^2 + U_p^2}, \tag{41}$$

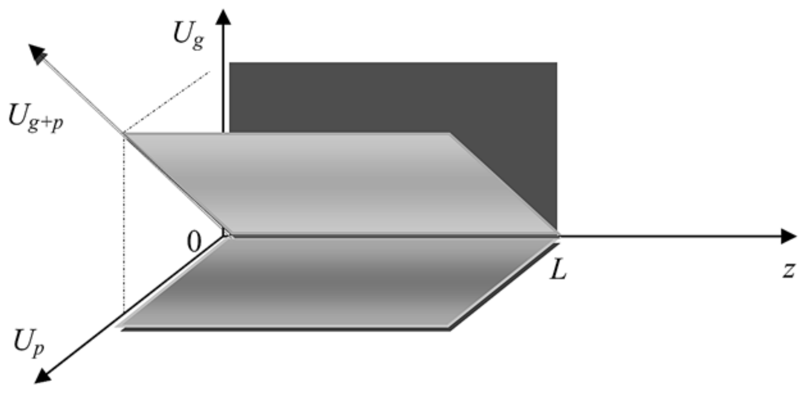

Fig. 2 – The potential energy U_{g+p} of interaction between photons and a waveguide filled with a lossless plasma. L is here the plasma filled waveguide length.

Notice that the waveguide potential energy U_g and that of the plasma U_p are out of phase by a phase factor $e^{i\pi}$ (Fig. 2).

Finally, the results concerning the photon propagation through a transparent, non-dissipative dielectric medium can be summarized as follows:

– the photon energy remains unchanged by the passage from a medium of refractive index n_1 in a medium with refractive index $n_2 \neq n_1$;

– the photon movement through a medium with refractive index n is governed by the variational principle of the relativistic analytical mechanics.

$$E = \vec{v}\vec{p} - L, \qquad L \to 0, \tag{42}$$

where E is the photon energy, $\vec{p}$ is its impulse, $\vec{v}$ is its propagation velocity and L is its Lagrange function. However, for the photon $L \to 0$ at the same time with the particle rest mass. Thus, the photon energy becomes $E = \vec{v}\vec{p} = \frac{c}{n} p = \hbar\omega$ (considering the vectors $\vec{v}$ and $\vec{p}$ as having the same orientation), in agreement with formula (37) which requires the introduction of a scalar potential energy, both Lorentz invariant and imaginary.

6. THE SYMMETRY PROPERTIES OF THE PHOTON WAVE FUNCTION

Let's consider in continuation the symmetry properties of the photon wave function in (plasma and dielectric) media. Thus, the wave function Ψ satisfies simultaneously the equation (20) (of Schrödinger-type), the equation of conservation of the probability and of the energy (26), and equation (38) of the generalized D'Alambert type (with the "potential" of the medium). The properties which we shall refer to devolve from the quality of $\vec{Q}$-vector (hexavector) of $\vec{\Psi}$. Generally, the $\vec{Q}$-vectors represent an alternate modality of the foundation of Maxwell-Lorentz electrodynamics, which is rather based on the Plucker-Cayley hexa-vectorial geometry than on the Poincaré-Minkowski tetra-dimensional geometry [6, 7].

The following advantages of this new thinking are emphasized here:

a) The formulation of electrodynamics in homogeneous fields (with ε, μ as constants of material) [1] and

b) The formulation of the Schrödinger-type of the refractive index problems (following an idea of Heisenberg [18].

A specific characteristic of $\vec{Q}$-vectors is the similarity between an infinitely slow rotation and an infinitely slow Lorentz translation [3]. In contrast to a finite roto-translation, the wave function Ψ obeys the law

$$\vec{\Psi}' = (1+\vec{q}^{\,2})^{-\frac{1}{2}}\left(\vec{\Psi}' + \vec{q}\times\vec{\Psi}\right) + \left[1-(1+\vec{q}^{\,2})^{-\frac{1}{2}}\right]\frac{(\vec{q}\cdot\vec{\Psi})\vec{q}}{\vec{q}^{\,2}},$$
$$\vec{q} = \vec{\omega} + i\vec{\beta}, \tag{43}$$

where $\vec{\omega}$ marks a pure rotation and $\vec{\beta} = \frac{\vec{v}}{c}$. By separating

$$\vec{\Psi} = \mathrm{Re}\,\vec{\Psi} + i\,\mathrm{Im}\,\vec{\Psi} \tag{44}$$

the transformation law (43) gives the invariants

$$(\mathrm{Re}\,\vec{\Psi})^2 - (\mathrm{Im}\,\vec{\Psi})^2 = I_1 \quad \text{(Larmor)},$$
$$(\mathrm{Re}\,\vec{\Psi})\cdot(\mathrm{Im}\,\vec{\Psi}) = \tilde{I}_2. \tag{45}$$

The invariant I_1 has been used by Joseph Larmor for the Minkowski-invariant formulation of the linear electrodynamics (Maxwell-Lorentz). The pseudo-invariant $\tilde{I}_2$ is used in various schemes proposed for the non-linear electrodynamics. We can conclude, therefore, that starting from the wave function $\vec{\Psi}$, we have a theoretical basis to cover not only the Maxwell-Lorentz electrodynamics in vacuum and in other homogeneous media

$$\vec{\Psi} = \frac{|c|}{\sqrt{8\pi}}\left(\sqrt{\varepsilon}\vec{E} + i\sqrt{\mu}\vec{H}\right). \tag{46}$$

but various relativistic variants of non-linear electrodynamics (Born, Infeld, Schrödinger, etc) as well.

The spin concept implied in our theoretical approach contains two distinct mathematical formulations.

a) an "electromagnetic spin" (see the matrix operators 17) and

b) a "quantum spin" (see the matrix operators 74). It is now opportune to introduce the notations

$$\vec{\mathcal{S}} = A\vec{i}_1 + B\vec{i}_2 + C\vec{i}_3 \qquad \text{(for the elctromagnetic spin)},$$
$$\vec{S} = \mathcal{A}\vec{i}_1 + \mathcal{B}\vec{i}_2 + \mathcal{C}\vec{i}_3 \qquad \text{(for the quantum spin)}, \tag{47}$$

The self-ortogonality conditions (66) lead to the spin algebra

$$\begin{array}{ll} BC - CB = iA & \mathcal{BC} - \mathcal{CB} = i\mathcal{A} \\ CA - AC = iB & \mathcal{CA} - \mathcal{AC} = i\mathcal{B} \\ AB - BA = iC & \mathcal{AB} - \mathcal{BA} = i\mathcal{C} \\ \text{(electromagnetic spin)} & \text{(quantum spin).} \end{array} \tag{48}$$

The algebra is formally the same, irrespectively of the manner used for inserting the concept. Further on, we may express the two spin algebras in a different (but equivalent way) by resorting exclusively to real matrices, denoted with common (latin and greek) letters

$$\begin{aligned} &A = ai, \quad B = bi, \quad C = ci, \\ &\mathcal{A} = \alpha, \quad \mathcal{B} = \beta i, \quad \mathcal{C} = \gamma. \end{aligned} \tag{49}$$

Now, a clear distinction between the two algebras is revealed, pointing out their belong to distinct physical objects

$$\begin{aligned} &bc - cb = a, \qquad \beta\gamma - \gamma\beta = \alpha, \\ &ca - ac = b, \qquad \gamma\alpha - \alpha\gamma = -\beta, \\ &ab - ba = c, \qquad \alpha\beta - \beta\alpha = \gamma. \end{aligned} \tag{50}$$

While the left placed algebra still preserve the cyclic rule of composition, the right placed algebra is qualitatively different (based rather on combinations than on cyclic permutations). The conclusion is that the goings over from electrodynamic spin to the quantum spin is not possible resorting exclusively to the q-vectorial transformation. It is necessary to perform a "rotation", but not in the real (x, y, z) space. The rotation should be in the complex three-dimensional space of the wave functions, which is of the Hilbert type.

Starting from the general roto-translation transformation law (43), two important particular transforms are revealed, namely

$$\begin{cases} \text{a) } \vec{\beta} = 0, \quad \vec{\omega} \neq 0 \quad \text{(pure rotation movement)} \\ \text{b) } \vec{\beta} \neq 0, \quad \vec{\omega} = 0 \quad \text{(inertial Lorentz movement).} \end{cases} \tag{51}$$

Explicitly, these transforms become

$$\begin{aligned} &\vec{\Psi}'_{\omega} = \eta\left\{\vec{\Psi} + \vec{\omega}\times\vec{\Psi} + \frac{\eta}{\eta+1}(\vec{\omega}\times\vec{\Psi})\vec{\omega}\right\}, \qquad \eta = (1+\vec{\omega}^2)^{-\frac{1}{2}}, \\ &\vec{\Psi}'_{\beta} = \gamma\left\{\vec{\Psi} + i\vec{\beta}\times\vec{\Psi} - \frac{\gamma}{\gamma+1}(\vec{\beta}\cdot\vec{\Psi})\vec{\beta}\right\}, \qquad \gamma = (1-\vec{\beta}^2)^{-\frac{1}{2}}. \end{aligned} \tag{52}$$

Using the formal transform [5, b]

$$(\vec{\omega} \to i\vec{\beta},\ \eta \to \gamma) \quad \text{or} \quad \left(\vec{\beta} \to \frac{1}{i}\vec{\omega},\ \gamma \to \eta\right), \tag{53}$$

the pure rotation transform goes into a pure Lorentz translation (and conversely). Using further the matrix notation instead of the vector (Cartesian) one, the wave function becomes

$$\begin{Vmatrix} \Psi'_1 \\ \Psi'_2 \\ \Psi'_3 \end{Vmatrix}_{\omega} = \Lambda(\omega)\begin{Vmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \end{Vmatrix}, \qquad \begin{Vmatrix} \Psi'_1 \\ \Psi'_2 \\ \Psi'_3 \end{Vmatrix}_{\beta} = \Lambda(\beta)\begin{Vmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \end{Vmatrix}, \tag{54}$$

where

$$\Lambda(\omega)=\left\|\begin{matrix} \eta+\frac{\eta^2}{\eta+1}\omega_1^2, & -\omega_3\eta+\frac{\eta^2}{\eta+1}\omega_1\omega_2, & \omega_2\eta+\frac{\eta^2}{\eta+1}\omega_1\omega_3 \\ \omega_3\eta+\frac{\eta^2}{\eta+1}\omega_1\omega_2, & \eta+\frac{\eta^2}{\eta+1}\omega_2^2, & -\omega_1\eta+\frac{\eta^2}{\eta+1}\omega_2\omega_3 \\ -\omega_2\eta+\frac{\eta^2}{\eta+1}\omega_1\omega_3, & \omega_1\eta+\frac{\eta^2}{\eta+1}\omega_2\omega_3, & \eta+\frac{\eta^2}{\eta+1}\omega_3^2 \end{matrix}\right\|, \tag{55}$$

$$\Lambda(\beta)=\left\|\begin{matrix} \gamma\left(1-\frac{\gamma}{\gamma+1}\beta_1^2\right), & \gamma\left(-i\beta_3-\frac{\gamma}{\gamma+1}\beta_1\beta_2\right), & \gamma\left(i\beta_2-\frac{\gamma}{\gamma+1}\beta_1\beta_3\right) \\ \gamma\left(i\beta_3-\frac{\gamma}{\gamma+1}\beta_2\beta_1\right), & \gamma\left(1-\frac{\gamma}{\gamma+1}\beta_2^2\right), & \gamma\left(-i\beta_1-\frac{\gamma}{\gamma+1}\beta_2\beta_3\right) \\ \gamma\left(-i\beta_2-\frac{\gamma}{\gamma+1}\beta_3\beta_1\right), & \gamma\left(i\beta_1-\frac{\gamma}{\gamma+1}\beta_3\beta_2\right) & \gamma\left(1-\frac{\gamma}{\gamma+1}\beta_3^2\right) \end{matrix}\right\|. \tag{56}$$

The symmetry properties of matrices $\Lambda(\omega)$ are

$$\Lambda^{-1}(\omega)=\tilde{\Lambda}(\omega)=\Lambda(-\omega), \qquad \tilde{\Lambda}(\omega)\Lambda(\omega)=\hat{I}. \tag{57}$$

With their help we conclude that $\Psi^+\cdot\Psi$ and $\Psi^+\vec{S}\Psi$ from Eq. (26) reveal the invariance properties required by the equation of conservation of the photon localization probability, namely

$$\Psi_\omega^{+'}\Psi'_\omega=\Psi_\omega^+\left[\tilde{\Lambda}(\omega)\Lambda(\omega)\right]\Psi_\omega=\Psi_\omega^+\Psi_\omega, \qquad \tilde{\Lambda}(\omega)\Lambda(\omega)=\hat{I}, \tag{58}$$

$$\Psi_\omega^{+'}\tilde{\Lambda}(\omega)\left[\Lambda(\omega)\vec{S}\tilde{\Lambda}(\omega)\right]\Lambda(\omega)\Psi_\omega, \qquad \vec{S}'=\Lambda(\omega)\vec{S}\tilde{\Lambda}(\omega). \tag{59}$$

The results expressed by the equations (57–59) are valid in a reference frame to be found in a pure rotation with respect to the propagation medium (transparent dielectric or plasma).

The Lorentz transforms are compatible with Eq. (38) if the following two conditions are fulfilled:

a) $\tilde{\Psi}\Psi$ is Larmor invariant of the electromagnetic field;

b) The wave equation has the form $O_L=0$, where O_L is a Lorentz invariant operator. The determination of the form of O_L on the basis of the hypothesis of the field U, produced by the propagation medium, leads to some deviations in the description of the quantum behavior of the photon in the considered medium. Such a peculiarity is the fact that Ψ is an eigen - function of the energy and of $\vec{S}^2$, but not of S_3.

Coming back to the equation (38), it comes out that the requirement of Lorentz invariant reduces to the condition

$$K = 4 - \text{dimensional invariant} \tag{60}$$

In its turn, this condition splits off in other two conditions, namely

$$\omega = \omega_0, \quad n = 4 - \text{dimensional scalar} \tag{61}$$

where ω_0 is the frequency in the rest frame with respect to the propagation medium. But ω_0 can be expressed by the frequency ω in the moving frame with the velocity $\vec{v} = c\vec{\beta}$ and under the angle $\sphericalangle(\vec{\beta}, \vec{p})$, that is

$$\omega_0 = \omega \frac{(1-\vec{\beta}^2)^{1/2}}{1+\beta\cos\theta}. \tag{62}$$

Consequently, Eq. (38) in the Lorentz – invariant form becomes

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Psi + \alpha\left\{\frac{\omega}{c}\left|n^2-1\right|^{\frac{1}{2}}\frac{\sqrt{(1-\vec{\beta}^2)}}{1+\beta\cos\theta}\right\}\psi = 0, \tag{63}$$

where $\alpha = -1$ or $+1$, as in Eq. (38). Finally, the relativistic expression of the potential energy of the medium-photon system is the following

$$U_p = \hbar\omega(1-n^2)^{\frac{1}{2}}\frac{\sqrt{(1-\vec{\beta}^2)}}{1+\beta\cos\theta}, \quad n<1, \quad \text{for plasma,} \tag{64}$$

$$U_d = i\hbar\omega(n^2-1)^{\frac{1}{2}}\frac{\sqrt{(1-\vec{\beta}^2)}}{1+\beta\cos\theta}, \quad n>1, \quad \text{for dielectric transparent medium}. \tag{65}$$

The photon propagation through material media (plasma or dielectric) implies directly (by the equations 20, 26) or indirectly (by the equations 63, 64, 65) the existence of the photon spin and of mathematical symmetries specific to spin algebra. The most general definition of the matrix-operator of the spin $\vec{S}$ is [22]

$$\begin{aligned} &\vec{S}\times\vec{S} - i\vec{S} = 0, \\ &\vec{S} = A\vec{i}_1 + B\vec{i}_2 + C\vec{i}_3. \end{aligned} \tag{66}$$

By introducing the expression of the operator $\vec{S}$ in the terms of the (A, B, C) operators in the first equation (66) (expressing the auto – perpendicularity of the spin) we get the condition

$$\{B\cdot C - C\cdot B - iA\}\vec{i}_1 + \{C\cdot A - A\cdot C - iB\}\vec{i}_2 + \{A\cdot B - B\cdot A - iC\}\vec{i}_3 = 0, \tag{67}$$

which should by identically satisfied. From here it results the matrix equations

$$B \cdot C - C \cdot B - iA = 0, \quad C \cdot A - A \cdot C - iB = 0, \quad A \cdot B - B \cdot A - iC, \tag{68}$$

where

$$A = \begin{Vmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{Vmatrix}, \quad B = \begin{Vmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{Vmatrix}, \quad C = \begin{Vmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{Vmatrix}. \tag{69}$$

Denoting by an accent the matrices transformed by rotation, that is

$$A' = \Lambda(\omega) A \tilde{\Lambda}(\omega), \quad B' = \Lambda(\omega) B \tilde{\Lambda}(\omega), \quad C' = \Lambda(\omega) C \tilde{\Lambda}(\omega), \tag{70}$$

$\tilde{\Lambda}$ being the transposed matrix of the matrix Λ, it comes out the composition conservation law of the algebra of spin 1, namely

$$B' \cdot C' - C' \cdot B' - iA' = 0, \quad C' \cdot A' - A' \cdot C' - iB' = 0, \quad A' \cdot B' - B' \cdot A' - iC' = 0. \tag{71}$$

The justification for the use of the matrices (69) for the photon spin ($S = 1$) is emphasized by the invariance of the length of the "spin vector" by a spatial rotation in E_3, that is

$$\frac{1}{2}(A^2 + B^2 + C^2) = \frac{1}{2}(A'^2 + B'^2 + C'^2) = 1. \tag{72}$$

We point out that an expression of the propagation equation of the photon through a material non-dissipative medium more general than equation (38) can yet be obtained by the addition of a coupling between the scalar interaction field $(\vec{E}_S, \vec{H}_S)$ and the spin $\vec{S}$ [23], that is

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathcal{F} + \alpha\left(\frac{U}{\hbar c} + \frac{m_0 c}{\hbar}\right)^2 \mathcal{F} - \frac{2q_S}{i\hbar c}\vec{S}\left(\vec{E}_S + i\vec{H}_S\right)\mathcal{F} = 0, \tag{73}$$

$$\vec{E}_S = \frac{1}{\left(m_0 c + \frac{U}{c}\right)}\left(\frac{1}{q_S}\nabla U p_4 - \frac{1}{c q_S}\frac{\partial U}{\partial t}\vec{p}\right),$$

$$\vec{H}_S = \frac{1}{\left(m_0 c + \frac{U}{c}\right)}\left(\frac{1}{q_S}\nabla U \times \vec{p}\right), \quad p_4 = \frac{i}{c}\mathcal{E}, \quad \mathcal{E} = \text{ the energy operator.}$$

In contrast to Ψ of Eq. (38), $\mathcal{F}$ of Eq. (73) contains also a field regularization, has a hermiticity transform ensuring the reality of the fields, and eliminates the divergences. Here $\alpha = \mp 1$ (as in equation 38), $\vec{v}$ is the photon velocity in the medium, $\vec{S}$ is the spin vector matrix (in the representation in which both $\vec{S}^2$ and S_3 are simultaneously diagonalized matrices), Ψ is the column matrix of the wave function (of Q-vector type), and U the Lorentz invariant scalar energy of interaction between the photon and the propagation medium. The expression of the spin matrix vector $\vec{S}$ is [24]

$$\vec{S} = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \vec{i}_1 + \begin{bmatrix} 0 & -\frac{i}{\sqrt{2}} & 0 \\ \frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{2}} \\ 0 & \frac{i}{\sqrt{2}} & 0 \end{bmatrix} \vec{i}_2 + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \vec{i}_3. \tag{74}$$

The demonstration of the Lorentz invariance of equation (73) can be made with the help of the method of infinitely slow transforms [23b]. For this purpose, let us denote by $\vec{\mathcal{S}}$ the spin 1 operator defined by Eq. (17) and by $\vec{S}$ the spin 1 operator defined by Eq. (74). The difference between the two definitions resides in the fact that $\vec{\mathcal{S}}$ has "zero trace", while $\vec{S}$ is built up so that S_3 is diagonal with the elements (+1, 0, –1). The connection between the two definitions is given by the unitary transform [24]

$$\vec{S} = \hat{U}\vec{\mathcal{S}}\hat{U}^{-1}, \quad \hat{U}^{+} = \hat{U}^{-1}, \quad \hat{U}^{+}\hat{U} = I, \tag{75}$$

where

$$\hat{U} = \begin{Vmatrix} -\frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \end{Vmatrix}. \tag{76}$$

Denoting by $\mathcal{F}$ the wave function in the representation in which S_3 is diagonal, we have

$$\mathcal{F} \equiv \begin{Vmatrix} \mathcal{F}_x \\ \mathcal{F}_y \\ \mathcal{F}_z \end{Vmatrix} = \begin{Vmatrix} \frac{1}{\sqrt{2}}(i\Psi_y - \Psi_x) \\ \Psi_z \\ \frac{1}{\sqrt{2}}(i\Psi_y + \Psi_x) \end{Vmatrix}, \quad \text{that is} \quad \mathcal{F} = \hat{U}\Psi. \tag{77}$$

Using the new functions, the equation (26) becomes an obvious quantum expression where, by contrast to Eq. (72), the diagonalization of S_3 is rather obtained by a rotation in the "spin space" than by a rotation in the real space E_3. Further we have

$$\frac{\partial}{\partial t}(\mathcal{F}^{+}\mathcal{F}) + \nabla \cdot \{ v\mathcal{F}^{+}\vec{S}\mathcal{F} \} = 0, \tag{78}$$

so that the condition of null divergence $\nabla \cdot \vec{\Psi} = 0$ from Eq. (15) now becomes

$$\left(-\frac{1}{\sqrt{2}} \frac{\partial}{\partial x^1} - \frac{i}{\sqrt{2}} \frac{\partial}{\partial x^2} \right) \mathcal{F}_1 + \frac{\partial}{\partial x^3} \mathcal{F}_2 + \left(+\frac{1}{\sqrt{2}} \frac{\partial}{\partial x^1} - \frac{i}{\sqrt{2}} \frac{\partial}{\partial x^2} \right) \mathcal{F}_3 = 0, \qquad (79)$$

so that

$$-\frac{1}{\sqrt{2}} \frac{\partial}{\partial x^1} \operatorname{Re} \mathcal{F}_1 + \frac{1}{\sqrt{2}} \frac{\partial}{\partial x^2} \operatorname{Im} \mathcal{F}_1 + \frac{\partial}{\partial x^3} \operatorname{Re} \mathcal{F}_1 + \frac{1}{\sqrt{2}} \frac{\partial}{\partial x^1} \operatorname{Re} \mathcal{F}_3 + \frac{1}{\sqrt{2}} \frac{\partial}{\partial x^2} \operatorname{Im} \mathcal{F}_3 = 0$$

$$-\frac{1}{\sqrt{2}} \frac{\partial}{\partial x^1} \operatorname{Im} \mathcal{F}_1 - \frac{1}{\sqrt{2}} \frac{\partial}{\partial x^2} \operatorname{Re} \mathcal{F}_1 + \frac{\partial}{\partial x^3} \operatorname{Im} \mathcal{F}_1 + \frac{1}{\sqrt{2}} \frac{\partial}{\partial x^1} \operatorname{Im} \mathcal{F}_3 - \frac{1}{\sqrt{2}} \frac{\partial}{\partial x^1} \operatorname{Re} \mathcal{F}_3 = 0.$$

This condition should necessarily accompany Eq. (20) in the representation with diagonal S_3, that is

$$-\frac{\hbar}{i} \frac{\partial}{\partial t} \mathcal{F} = v\left(\vec{S} \cdot \vec{p}\right) \mathcal{F}. \qquad (80)$$

Under these conditions Eq. (73), which is Lorentz invariant and in agreement with the experiments of Fizeau and Cerenkov [5b], can be considered not only equivalent with the Helmholtz equation (12), but also equivalent with the limit for $m_0 \to 0$ of a particle of spin 1 embedded in a combined vectorial and scalar interaction [23]. The spatial averaging of the scalar interaction becomes macroscopically manifest by the index of refraction. If the spatial and the temporal gradients of the refraction index are different from zero, then we must have a coupling between the photon spin with the photon propagation direction through the medium, even if the photon rest mass vanishes!

In the spirit of the present paper there were made several aplications in optics [19–21], in electronics [16, 17, 25], and in nuclear physics [26].

CONCLUDING REMARKS

The purpose of this study was to describe in quantum terms, through a specific continuous interaction between a single photon and a homogeneous, infinite medium (fully ionized plasma or a transparent dielectric) the energy balance photon – medium during the light travelling, leading to a certain wave equation for the interacting photon. Although inertial mass and electrical charge are vanishing physical parameters, to explain the interaction with the surrounding matter it is necessary to assume a certain photon – medium interaction energy, macroscopically materialized by the existence of the refractive index and of the Helmholtz type equation as an adequate tool for controlling the traveling process.

The nature of the interaction turns out to be of a scalar type, and the photon wave equation appears as the limiting case, for vanishing rest mass, of a spin 1 particle submitted to both scalar and vectorial fields.

The ireducible spinor concept is however circumvented, by resorting to the Q-vectorial (be hexa-vectorial) approach due (independently) to C. G. Bollini (1961), J. S. Dowker (1966) and Nicholas Ionesco-Pallas (1967) [23].

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APPENDIX
The passage from the electrodynamics (traceless) spin to the quantum spin (with diagonalized S_3)

By a rotation in the 6-dimensional Plucker & Cayley space it is possible to achieve simultaneously two requirements, namely

– Invariance of the quantity $\Psi^{*}\Psi$ and preservation of spin 1 algebra

– Diagonalization of the component S_3 (of the quantum spin)

We emphasize the fact that the "spin rotation" does not represent a rotation in the current sense because it does not imply the time. Specifically, this is achieved by a special transform (L. Foldy) by which a 4-vector Ψ is decomposed into a scalar (corresponding to a particle of spin 0) and a spatial vector (corresponding to a particle of spin 1), while the particle de spin 0 is eliminated [24, 27].

Actually we have to solve the matrix equation

$$\vec{S}U = U\vec{\mathcal{S}}, \tag{A1}$$

where $\vec{\mathcal{S}}$ is the electrodynamic spin given in (17) and $\vec{S}$ is the quantum spin given in (74). Both these spins satisfy the same algebra

$$\vec{\mathcal{S}}\times\vec{\mathcal{S}} = i\vec{\mathcal{S}}, \qquad \vec{S}\times\vec{S} = i\vec{S}. \tag{A2}$$

Denoting the transformation matrix between the two kinds of spin by

$$U = \begin{Vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{Vmatrix} \tag{A3}$$

the equation (A1) offers 9 vectorial equalities that is 27 scalar equalities. The 9 elements of the matrix U can be expressed in terms of a single element which we take conventionally by a_1, so that finally we have

$$a_1 = a_1, \quad a_2 = -ia_1, \quad a_3 = 0,$$
$$a_4 = 0 \quad a_5 = 0 \quad a_6 = -\sqrt{2}a_1, \tag{A4}$$
$$a_7 = -a_1, \; a_8 = -ia_1, \quad a_9 = 0.$$

The determination of the element a_1 can be achieved from the condition of hermiticity and unitarity of U

$$U^{+}U = 2|a_1|^2 I,$$

from where we get $a_1 = \frac{1}{\sqrt{2}}(\cos\alpha + i\sin\alpha)$. For $\alpha = \pi + \delta$, δ being an arbitrary phase, we get the expression (76) of $\hat{U}$ multiplied by the arbitrary phase factor $e^{i\delta}$.

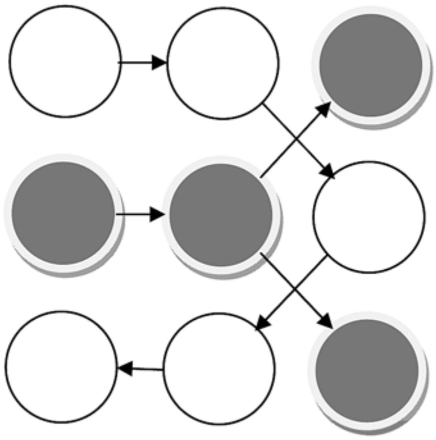

Fig. 3 – The *U* matrix diagram, where  is the null element.

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