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THE ENTROPY OF HOT QCD AT LARGE N_f: SUCCESSFULLY TESTING WEAK COUPLING TECHNIQUES

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Abstract. It has been known for some time that the entropy of hot QCD is well reproduced by weak coupling techniques. These do not identify with perturbation theory, known to have poor convergence properties, but involve resumming the physics of hard thermal loops in the non-perturbative Φ-derivable two-loop approximation. We test this approximation scheme in the limit of large flavor number (N_f) where the exact result can be calculated for a wide range of couplings for which the influence of the Landau pole is negligible. Using full momentum-dependent fermionic self-energies instead of the previously only known weighted average value to order g^3, the exact result for the entropy can be remarkably well reproduced by the HTL resummed theory for a natural choice of the renormalization scale, and this up to large values of the coupling. This gives confidence in the reliability of weak coupling techniques when applied to the thermodynamics of QCD, at least for temperatures $T \geq 3T_c$.

Key words: OCD, entropy, number of flavors, heavy ion collisions, perturbation theory.

INTRODUCTION

The entropy of hot QCD is well reproduced for temperatures $T \geq 3T_c$ by the non-perturbative Φ-derivable two-loop approximation [1]. Its success has been established by comparison to lattice simulations [2]. Here we want to present an independent test [3] of this approximation scheme in the limit of large flavor number N_f [4].

Approximations based on the Φ functional are constructed from the two-particle-irreducible (2PI) skeleton expansion [5] where the thermodynamic potential is expressed in terms of dressed propagators according to

$$\Omega[G,S]/T = \frac{1}{2}\mathrm{Tr}\log G^{-1} - \frac{1}{2}\mathrm{Tr}\,\Pi G - \mathrm{Tr}\log S^{-1} + \mathrm{Tr}\,\Sigma S + \Phi[G,S], \quad (1)$$

where "Tr" refers to full functional traces and $\Phi[G,S]$ is the sum of 2PI "skeleton" diagrams. The self-energies $\Pi[G,S] = G^{-1} - G_0^{-1}$ and $\Sigma[G,S] = S^{-1} - S_0^{-1}$ are determined by the stationarity property $\delta\Omega[G,S]/\delta G = 0 = \delta\Omega[G,S]/\delta S$. For the entropy $\mathcal{S} = (\partial P/\partial T)_\mu$ with $P = -\Omega/V$ one obtains [1]:

$$\begin{aligned}\mathcal{S} = &-\mathrm{tr}\int_K \frac{\partial n(k_0)}{\partial T}\Big[\,\mathrm{Im}\log G^{-1} - \mathrm{Im}\,\Pi\,\mathrm{Re}\,G\Big] \\ &-2\,\mathrm{tr}\int_K \frac{\partial f(k_0)}{\partial T}\Big[\,\mathrm{Im}\log S^{-1} - \mathrm{Im}\,\Sigma\,\mathrm{Re}\,S\Big].\end{aligned} \tag{2}$$

In the large N_f limit for QCD [4], the effective coupling $g_{\mathrm{eff}}^2 = g^2 N_f/2$ is kept finite as the limit $N_f \to \infty$ is taken. The thermodynamic potential to order N_f^0 and to all orders in g_{eff} is then given by a set of ring-diagrams which can be resummed. Large-N_f QCD is no longer asymptotically free. Its renormalization scale dependence is determined (non-perturbatively) by the one-loop beta function $\beta(g_{\mathrm{eff}}^2) = g_{\mathrm{eff}}^4/(6\pi^2)$ which implies a Landau singularity. The two-loop entropy formula (2) reduces in the large N_f limit to

$$S - S_0 = -\mathrm{tr}\int_K \frac{\partial n(k_0)}{\partial T}\Big[\,\mathrm{Im}\log G^{-1}/G_0^{-1} - \mathrm{Im}\,\Pi\,\mathrm{Re}\,G\Big] - 2\,\mathrm{tr}\int_K \frac{\partial f(k_0)}{\partial T}\Big[\,\mathrm{Re}\,\Sigma\,\mathrm{Im}\,S_0\Big]. \tag{3}$$

Using the 1-loop self-energy Π, this formula actually reproduces the large N_f entropy to all orders in g_{eff}. The fermionic self-energy Σ from this formula only has to be evaluated on the light-cone. The bare self-energy Σ_{b} is given by

$$\Sigma_{\mathrm{b}}(K) = \Sigma_{\mathrm{th}} + \Sigma_{\mathrm{b,vac}} = -g^2 C_{\mathrm{F}} \sum\!\!\!\!\!\!\int_Q \gamma^\mu S_0(Q+K)\gamma^\nu G_{\mu\nu}(Q). \tag{4}$$

The thermal piece Σ_{th} of the q_0 integration is UV finite, while the real part of the vacuum piece $\mathrm{Re}\,\Sigma_{\mathrm{b,vac}}$ is in general logarithmically divergent, but finite on the light-cone. Because of the Landau pole Λ_L, we have to introduce a Euclidean invariant cutoff $\Lambda^2 < \Lambda_L^2$, and the analytic continuation of $\Sigma_{\mathrm{b,vac}}$ to the light-cone requires a deformation of the integration path [3]. The numerically demanding calculations have been performed on the ECT* Teraflop cluster.

HTL APPROXIMATION OF THE SELF-CONSISTENT ENTROPY

The HTL effective action [6] is an effective action for soft modes with energy scales $\sim gT$. Inserting HTL into the entropy formula (2) gives the correct g^2 contribution, but only a fraction of the plasmon term $\sim g^3$. The larger part of the plasmon term arises from *hard* momentum scales where HTL is no longer accurate, namely from corrections to the leading-order asymptotic masses

$$\hat{\Pi}_T(k_0=k)=\hat{m}_\infty^2=\frac{1}{2}\hat{m}_D^2, \qquad \hat{\Sigma}_\pm(k_0=\pm k)=\frac{\hat{M}_\infty^2}{2k}=\frac{\hat{M}^2}{k}. \tag{5}$$

The NLO corrections to the asymptotic thermal masses are nontrivial functions of the momentum,

$$\delta m_\infty^2(k)\equiv \mathrm{Re}\,\delta\Pi_\mathrm{T}(k_0=k), \qquad \delta M_\infty^2(k)\equiv \mathrm{Re}\,2k\,\delta\Sigma_+(k_0=k), \tag{6}$$

and involve only a single HTL propagator and no HTL vertices for hard external momentum. These expressions can be evaluated only numerically (NLO-HTL). However, the weighted averages (NLA) are determined to order g^3 [1].

As one can see in Fig. 1, the NLO-HTL-resummed result represents a considerable improvement over the HTL-resummed result truncated at order g_eff^3 for $g_\mathrm{eff}^2\geq 4$, which corresponds to $\hat{m}_D/T\geq 1$. Fig. 2 shows the results of a numerical calculation of the asymptotic thermal quark mass squared $M_\infty^2(k)\equiv 2k\,\mathrm{Re}\,\Sigma(k_0=k)$ for $g_\mathrm{eff}^2(\bar{\mu}_\mathrm{MS}=\pi T)=9$, normalized by T^2C_f/N_f. The exact (nonperturbative) result obtained in the large-N_f limit is given by the full lines. In Fig. 3 the exact and the NLO-HTL results for the entropy are compared to simpler approximations. It is remarkable that a full HTL resummation using

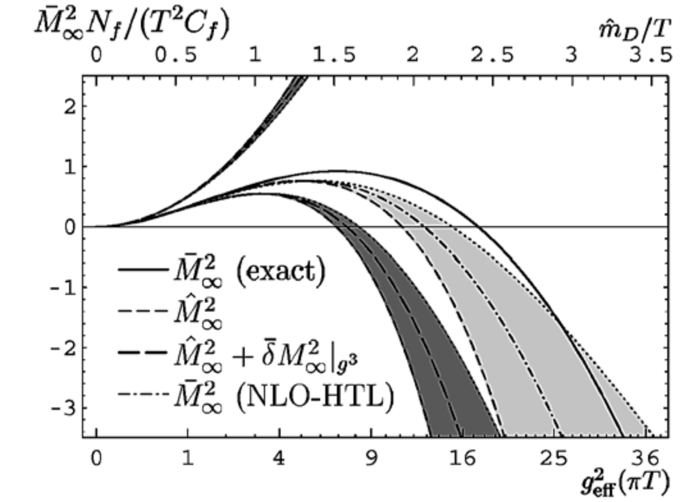

Fig. 1 – Comparison of the averaged asymptotic thermal quark mass squared, $\bar{M}_\infty^2$, in various approximations. In this and the following plots, the renormalization scale $\bar{\mu}_\mathrm{MS}$ is varied around the FAC-m scale by factors of 2.

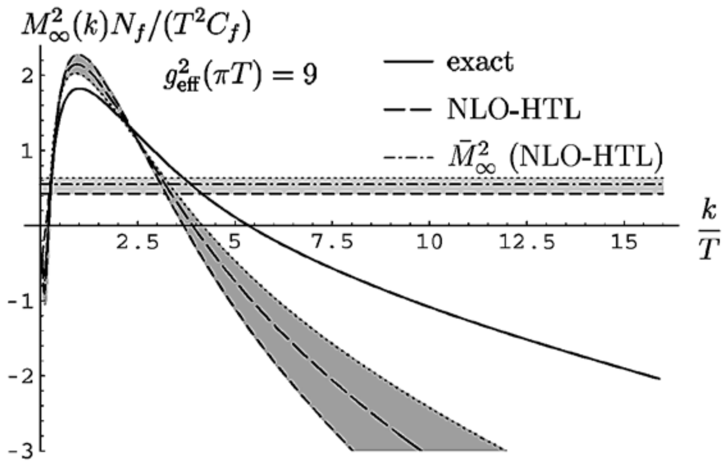

Fig. 2 – Asymptotic thermal quark mass squared as a function of k/T for $g^2_{\text{eff}}(\pi T) = 9$. The exact large- N_f result is compared to the NLO-HTL calculation and its relevant average value.

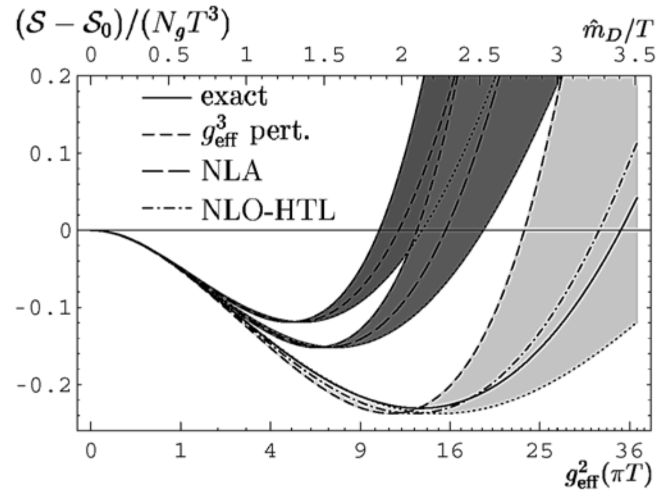

Fig. 3 – Entropy in the large-N_f limit, comparing the exact large-N_f result to the strictly perturbative expansion through order g^3_{eff}, the NLA result, and the NLO-HTL result.

momentum-dependent asymptotic quark masses can extend the validity of the approximation in the large N_f limit up to $\hat{m}_D/T \sim 2.5$. For the entropy of (small N_f) QCD we therefore expect likewise improvement once full HTL-resummed momentum-dependent asymptotic gluon masses have been implemented.

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